

**Expansion of Firms and Human Capital Accumulation by Training:  
A Growth Model for the Not-So-Growing**

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## **Expansion of Firms and Human Capital Accumulation by Training:**

### **A Growth Model for the Not-So-Growing**

This paper presents a novel theory of development. Endogenous growth literature analyzes the discovery of new knowledge, whereas the poorest among the less developed countries are unable to utilize existing knowledge. Capital may flow to these countries if productivity improves, but productivity depends on human capital. Human capital growth-literature addresses its accumulation through schooling, but it is accumulation through on-the-job training which is required in order to use available knowledge. Moreover, unlike schooling, on-the-job training requires existence of firms and their incentives to train, not a decision of the workers. This paper fills this gap in development theory by presenting a highly stylized general equilibrium endogenous development-growth model based on firms and their expansion through on-the-job training of managers. Training new managers increases the profitability of the firm, and since managers can eventually head new firms and train more managers, a development process is generated, a process in which output and productivity increase up to a full utilization of existing technology. Since managers and firms, both important for production, are scarce in poor countries, the model is suitable for development analysis. The paper therefore sheds light on the restraints to accumulation of human capital in less developed countries and the importance of firms' expansionary behavior to development.

#### **1. Introduction**

This paper presents a theory of development based on accumulation of human capital through on-the-job training. Since firms that can train and trained managers are both scarce in less developed countries, this paper enables understanding of the constraints to human capital accumulation and the importance of the firms' behavior to the development and catch up process. Endogenous growth literature deals with the invention of new knowledge, whereas in the less-developed countries the problem is how to utilize the available knowledge. The human capital literature analyzes the ability and educational choice of workers, ignoring the fact that using the available knowledge requires on-the-job training by firms as well, firms which are in short supply in less-developed countries. Therefore, the aforementioned literature is unsuitable for the understanding of the development problems of these countries and for planning appropriate policies. The study presented here develops a general

equilibrium endogenous development-growth model which is based on the training of managers by firms, whereas both managers and firms are a scarce resource in less developed countries and on the contribution of these managers to the production process. Thus, the model stresses the supply side restraints to accumulation of human capital in less developed countries and the importance of firms' expansion to development, thus enabling a better understanding of the development problems and helping reassess policies aimed at improving economic performance.

As noted by Easterly (2007), development policies have failed in helping most of the less developed countries. Policies have varied throughout the years, starting from raising investment rates, moving to structural changes and openness to trade, and later improving institutions, protecting property rights and lessening corruption. All policies resulted in no sustained development and occasional debt crises in many of these countries. Easterly (2006) rejects empirically the notions of poverty traps, big-pushes and takeoffs, and Kraay and Raddatz (2007) empirically reject low-technology-based and low-savings-based poverty traps, thus, leaving us with a gap in the development theory for these countries. So what prevents the development of those countries?

Early growth literature focused on capital as a source of development and growth (Solow, 1956), or at least of bounded growth. But although capital is in short supply in those countries, it can flow there from richer countries. However, as noted by Lucas (1990), it does not, probably due to lower level of human capital and its externalities, which results in lower productivity and worse technology. Endogenous growth literature (Romer, 1990; Grossman and Helpman, 1991; Aghion Howitt, 1992; and Aghion et al., 2001) focuses on the importance of R&D, the development of new knowledge and developing new or better products and thus improving the technology

as the cause for growth. However, this is not relevant for the development problem some of the poorest countries are facing – how to utilize the existing knowledge. Even the north-south literature (see, for example, Krugman, 1979; Acemoglu and Zilibotti, 2001; Chen and Puttitanun, 2005; and Glass and Wu, 2007) assumes the capability of imitation and absorption of technologies by the south, a capability which does not exist in these less developed countries. This study will therefore focus on the development phase in which the country tries to utilize the existing technology and once this is done the country actually moves to the growth phase described by the models above.

Another explanation for the inability to adopt and utilize advanced technologies is the lack of human capital. The importance of human capital has been expressed by Becker (1964), and its relation to development and growth has been extensively investigated, theoretically and empirically; see for example Lucas (1988), Romer (1989), Becker, Murphy and Tamura (1990), Stokey (1991), Glomm and Ravikumar (1992), Mankiw, Romer and Weil (1992), Galor and Tsiddon (1997), Barro (2001), Eicher and Gracia-Penalosa (2001) and Galor and Moav (2004). However, while the vast majority of this literature regards schooling as the equivalent of human capital and analyzes agents' demand for education, it ignores the fact that a substantial part of the training of managers, technicians, engineers and other professional workers is on-the-job training, a form of human capital that was identified as highly influential by Mincer (1962), estimating the costs and returns to experience and on-the-job training, and by Becker (1964). This form, which was recognized as having a similar level of importance as formal schooling, is crucial for utilizing the relatively abstract knowledge gained in the education system to absorption of advanced technologies, and differs substantially from schooling. While

schooling is a decision made and financed by the individual or the individual's parents, on-the-job training requires active participation of employers, the firms, firms which also incur most of the cost. Moreover, this type of training requires the existence of industrialized firms, and this may not be the case for the less developed countries. Shortage in firms creates a limited capacity for on-the-job training, thus it puts constraints on the potential of increasing this type of human capital and on the capacity of absorption of new technologies. Two exceptions within the literature offer on-the-job training as an alternative interpretation to their main line of modeling, but do so while ignoring its unique characteristics, and thus they are inappropriate for modeling on-the-job training. Galor and Tsiddon (1997) offer this interpretation as an alternative to schooling, and Lucas (1988) addresses a learning-by doing mechanism for human capital accumulation and gives it an alternative interpretation of on-the-job training, but both are ignoring the need for a trainer firm in such a model, the possible scarcity of these firms, their costs in training and their active decision whether to train or not.

Thus, this paper focuses on on-the-job training in an environment in which the technology and the production function is already given, and the economy needs to utilize it through the training of skilled managers (or entrepreneurs). The economy consists of two sectors, a traditional sector with fixed marginal productivity of labor and a technological-manufacturing sector. Every period, the economy is populated with a limited amount of managers (an amount which changes over time), each heading a firm (or a production unit) in the manufacturing sector.<sup>1</sup> The production function of each unit is given and demonstrates diminishing marginal productivity, which combined with the marginal productivity in the traditional sector and a

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<sup>1</sup> Similar role for the manager-entrepreneur appeared in Dias and McDermott (2006), but without any training necessary for this role, thus ignoring the aspect of the development problem approached here.

competitive labor market, sets the employment and output of every such unit. Every firm (or unit) can decide, however, on expanding by training managers (at a cost that demonstrates increasing marginal cost) and thus opening more production units that will enjoy the same production function with decreasing marginal productivity, increasing its employment, output and profits. In the next period these newly trained managers, which are heading firms-production units (through a competitive manager's labor market) can train new managers, thus increasing the amount of production units, each enjoying the manufacturing sector's production technology. Eventually, once the traditional sector disappears (second phase hereinafter), more managers and production units reduce the employment per unit thus increasing the marginal productivity of the workers. The economy is in general equilibrium every period, such that consumption equals the output.

Thus, the model presents a development process in which profits maximizing and on-the-job training expands the manufacturing sector and increases the marginal productivity in this sector over time. Output is also increasing throughout the process, since in the first phase larger share of the economy is using the advanced manufacturing technology every period, and in the second phase the technological sector is using this technology more efficiently every period. As this is a model of development and catch-up, the economy eventually utilizes the technology efficiently. This is the phase where this model ceases to be relevant for that country, and future growth depends on new knowledge and technology, as in the endogenous growth theory. During the development phase described above, the model copes with the two main deficiencies in the literature regarding development – the need to apply the

existing technology in order to grow rather than developing a new one;<sup>2</sup> and, the shortage of trained labor required to do that, labor which requires on-the-job training, not just education.<sup>3</sup> Thus it shows that supply side restraints, the shortage in trained managers and training firms, may limit accumulation of human capital in less developed countries, and that firms' profit maximizing through expansionary behavior, and the profitability of expanding may be crucial for development. This also enables assessing various policies, see Discussion for elaboration.

The increase in the amount of production units in the model resembles the increase in the amount of firms or intermediate goods in the endogenous growth literature (Romer, 1990; and Grossman and Helpman, 1991). The difference lays within the mechanism of development in this study, which is training new managers rather than inventing new goods, or in an abstract interpretation training entrepreneurs which are in a short supply by existing firms in order to maximize firms' profits in this paper, rather than an unlimited stock of entrepreneurs which act on their own profit maximizing. Literature regarding firms, which owes a lot to the seminal work of Coase (1937), has long recognized the need for the training of managers as an engine for expansion of a single firm (Penrose 1955; Penrose, 1959; and Slater, 1980). However, this literature has not aggregated the individual firms into a general equilibrium framework where a firm can hire a manager, but managers are in limited supply in the economy as a whole, their wage may be affected by the aggregate activities of the firms etc., as is being done here. Contrary to the no-aggregation mentioned above, Lucas (1978), when considering managers, assumes a given supply

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<sup>2</sup> Hulten and Isaksson (2007), in an empirical investigation of 112 countries over 30 years, found indeed that efficiency of production, and not capital formation, is the dominant factor accounting for differences in development levels.

<sup>3</sup> Kosempel (2004) offers a two stages model, development by acquiring human capital followed by growth by technological innovation, but since the first phase is schooling-based, it suffers from the same shortfalls as all the human capital literature mentioned above.

of managerial skill in the population, ignoring the need for and the cost of training and the effect of the training on the supply managers. In the less developed countries, this quantity of managers or professional workers and the capacity for training these workers may be the binding constraint. While capital can flow to these places relatively easy (contingent on adopting modern technologies, good institutions etc., and keeping the assumption that trained workers and managers are required to efficiently utilize capital and labor), the skilled labor required for this is much less mobile and requires much larger incentives to attract from abroad, incentives that may make it unworthy to do so. Therefore, the internal training is an important aspect of the development.

Education, the common form of human capital found in the literature, may be required in order to be trained on-the-job. Based on costs, workers may acquire education in order to be trained, and thus the development process described may also include increased schooling. Although not explicitly modeled, this issue and the population which is heterogeneous with respect to ability, are discussed in sub-section 3.3. Cost sharing, workers paying the cost of their general training, appeared in Becker (1964), see also Hashimoto (1981). Although not explicitly in the model, including it would not alter the general form of the development process presented, see section 2 and sub-section 3.1 for elaboration.

Firm specific skills acquired by on-the-job training, which increases workers' productivity only within the firm, compared to general on-the-job training, giving general skills and increasing workers' productivity in every possible firm, has been presented by Becker (1964), see also Stevens (1994). The model presented in section 2 complies with general training (although the first period, in which the newly-trained manager must work for the training firm, may be regarded as a firm-specific training,

which later diffuses to general skills). Subsection 3.4 discusses firm specific skills, thus presenting multiple technological sectors and differentiated goods as well. The general form of the development process and catching up with existing technologies remain unchanged.

Kuznets (1955), in his presidential address to the American Economic Association, suggested that income inequality is generally rising in early stages of economic development, stages which are accompanied by urbanization, while in the latter stages of development inequality declines. Despite some debate, this inverted U-shaped relationship has gained empirical support, see for example Barro (2000) and a survey at Deutsch and Silber (2004). The model is consistent with the Kuznets hypothesis and the relevant empirical literature, generating an increase in inequality in the early stages of development since there are more and more managers with higher wage in the technological sector and generating a declining inequality in latter periods, as traditional sector vanishes, workers' wage increases and eventually all the population is trained. Moreover, assuming that the technological sector is concentrated in urban places, the model also generates the aforementioned urbanization. For positive but not too large externalities of managers, it also generates an increase in the relative wage of trained managers in the initial stages and a decrease in the latter stages, a phenomenon often traced throughout the evolvement of inequality (see Galor and Tsiddon 1996). This behavior is also consistent with Deutsch and Silber (2004) finding that a decline in the entrepreneurial income is an important factor in the declining inequality in latter periods, since managers in this model may also be viewed as entrepreneurs and their relative wage is indeed decreasing in these stages. It should be mentioned that inequality itself is not affecting the development in the model; it is a result of the development process.

The paper proceeds as follows: Section 2 presents the model. Section 3 analyzes the development path and presents possible extensions to the model. Section 4 presents the evolution of income inequality throughout the development path. Section 5 concludes.

## 2. The Model

Assume a closed economy with a population size of  $N$  infinitely lived agents. Each agent is working every period in the traditional sector as a worker in the manufacturing-technological sector or as a manager in that sector. Denote with  $F_t, L_t$  and  $M_t$  the quantities of workers in the traditional sector, workers in the technological sector and managers, such that

$$(1) \quad F_t + L_t + M_t = N .$$

Production of a single perishable good takes place in two sectors, traditional and technological, using labor only. The traditional sector exhibits a constant marginal productivity (which is normalized to 1). The technological sector is divided into production units (plants, firms etc.). Each one of those production units requires a manager in order to exist and has a production function in the form of

$$(2) \quad y = f(l)$$

where  $y$  is the output of the production unit and  $l$  is the number of employees including the manager,  $f(l)$  is increasing in  $l$  with decreasing marginal productivity, satisfying the Inada conditions and satisfying  $\frac{\partial f}{\partial l} > 1$  for  $l = 1$ , i.e. the marginal productivity of the first worker in the technological sector is greater than that of a

worker in the traditional sector.  $f(l) = Al^\alpha$ ,  $A > \frac{1}{\alpha}$  will serve as an example. Labor markets are competitive.

An alternative interpretation, which does not change the modeling (though may change the calculation of the growth rate), is of differentiated goods, where each production unit produces a different good. Since we are dealing with a model of adoption of exiting technologies, it does not have to perform R&D in order to do so but rather to choose a good that was already developed elsewhere. However, throughout the modeling we will stick to the original interpretation. Sub-section 3.4 discusses firm specific skills in a differentiated goods context.

Training managers is the heart of this development model. An agent can become a manager only through training in an existing firm-production unit. Each period each firm (a production unit headed by a pre-trained manager) can train new managers at a cost. The newly trained managers can immediately function as managers (i.e. to enable the current firm-unit to open more production units), although the new managers and the new production units cannot train more managers at the same period. In the current period, the new managers are obliged to work for that same firm, but in later periods they are free to leave or head new firm-production units that can train new managers. The cost  $c$  of training  $m$  new managers in a single production unit in period  $t$  is denoted by

$$(3) \quad c = c(m, M_{t-1})$$

where  $c(0, M_{t-1}) = 0$ ,  $\frac{\partial c}{\partial m} > 0$ ,  $\frac{\partial^2 c}{\partial m^2} > 0$ ,  $\frac{\partial c}{\partial m} \xrightarrow{m \rightarrow 0} 0$  and  $\frac{\partial^2 c}{\partial M_{t-1} \partial m} \leq 0$ . The first

four conditions resemble standard investment-cost function. The last condition measures the externalities of having more managers in the population on the training cost of new managers (one may consider, for example, establishing business schools

beyond a certain amount of managers in the population, such that the cost of training managers in the firms is decreasing; a similar assumption is common in the growth literature regarding the externalities of existing knowledge of the cost of developing new knowledge), where equality in the last condition means no positive externalities of the quantity of managers. The functional form  $c(m, M_{t-1}) = \frac{m^2}{M_{t-1}^\beta}$ ,  $0 \leq \beta \leq 1$  will serve as an example.

The firm can not charge managers for their training, i.e., they are paid their marginal productivity as workers (like every other worker; note that  $l$  includes the manager). The behavior described here may be regarded as cost sharing, since the average productivity of the newly trained manager is higher than that of a worker, although that manager is paid like a worker. However, due to the increasing marginal cost of training this is not the case with respect to marginal productivity.<sup>4</sup> Allowing firms to pay newly-trained managers less than their marginal productivity as workers would increase profitability of training and speed up development but would not change the essence of the process described here, see sub-section 3.1 for elaboration.

There are neither savings nor capital in this economy. Accordingly, all the agents consume their entire income every period. Note that I abstain from adding capital into the model, since this is not the focus of the development problem analyzed here. When production requires capital, it may flow in as needed if the country manages to adopt advanced technologies such that the productivity of this capital is high enough. Therefore, the focus is on utilizing the technology.

Every period existing managers (i.e. all those that worked as managers in the previous period, whether they were trained in that period or before) are running (and

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<sup>4</sup> Due to the increasing marginal cost of training, and since firm train managers till the marginal cost of training equals their excess productivity, newly trained managers' marginal productivity above their productivity as workers is 0.

working in) the firm-production units. Each firm decides how many managers to train (and therefore how many new units to open) and how many workers to hire. The rest of the population, if there is any, work in the traditional sector. Production and consumption take place, and we move to the next period.

### 3. Development Path

Prior to the development process all the output is coming from the traditional sector, which yields an output of  $N$  per period. Assume that at period 1 one manager has been introduced to the economy.

#### 3.1 First Phase

The first phase refers to the periods in which there are still workers in the traditional sector, i.e.  $F_t > 0$ . As long as there are workers in that sector, the quantity of workers in every production unit is set by equating the marginal productivity in the technological sector to 1, the marginal productivity in the traditional sector. The decreasing marginal productivity combined with Inada conditions ensure that this is achieved for a finite quantity of workers noted with  $\bar{l}$ . For the explicit example one gets  $\frac{\partial f(l)}{\partial l} = \alpha A l^{\alpha-1} = 1$ , and therefore  $\bar{l} = (\alpha A)^{1/(1-\alpha)}$ .

Each period each firm decides how many new managers to train. The profit of a certain firm which is training  $m$  new managers, excluding the wage of the already existing manager, is:

$$(4) \quad \pi(m) = (m+1)(f(l) - w(l-1)) - wm - c(m, M_{t-1})$$

where  $w$  is the wage of a worker,  $w=1$  as long as the traditional sector exists. Note that the profit is simply the output minus the wages of the workers, the newly trained managers (which are paid like workers) and the cost of training. The wage of the pre-

trained manager equals his marginal product, which is the mere existence of the firm, and therefore actually the whole profit of the firm including its new production units. Consequently, one may regard the manager as the owner-entrepreneur as well. Note also that since in future periods firms will have to pay the managers they are training in the current period a competitive wage which equals their marginal product, and since all the population is equally talented, people would agree to pay the firms in order to be trained. However, it is assumed that firms are not allowed to pay the managers they train less than a worker's wage (another minimum wage may also be assumed here), and that they can not force managers to stay within the firm (or to stay more than a given number of periods). Therefore, all the population would like to be trained, and firms arbitrarily choose who to train. See 3.3 for education and for agents which are heterogeneous with respect to ability.

**Proposition 1:** first phase growth rate of the technological sector depends on the externalities in management training, such that the growth rate of the sector is constant without externalities, and increases over time with positive externalities.

**Proof:** given the wage of the pre-trained manager, firms maximize the profits, (excluding that wage, see equation (4)) over  $m$ . Accordingly,

$$\frac{\partial \pi(m)}{\partial m} = f(l) - l - \frac{\partial c(m, M_{t-1})}{\partial m} = 0. \text{ Since } l = \bar{l}, m \text{ is set according to}$$

$$(5) \quad \frac{\partial c(m, M_{t-1})}{\partial m} = f(\bar{l}) - \bar{l},^5$$

which is actually equating the marginal cost of training one more manager to the marginal benefit – one more production unit minus the wage of its workers. Note that

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<sup>5</sup> The optimal  $m$  may be a fraction, while the quantity of managers or production units must be an integer. My interpretation here is that in this case some firms train the quantity of the integer below  $m$  and some the integer above, such that the average of new managers is  $m$ , or that in some of the periods a firm trains managers in the quantity of the integer below  $m$  and in some the integer above, such that the average is, again,  $m$ . Another interpretation, not taken here, is that all the firms choose the more profitable option among the integer below and the integer above. Thus if  $m$  is below 1 and a quantity of 0 is more profitable than 1, one gets a trivial case of no development.

when (5) holds the marginal productivity of a newly trained manager beyond his productivity as a worker is 0; the firm does make a positive profit, however, from all the newly-trained managers besides the last one. Note also that allowing firms to charge managers a constant price (or a price bounded by their wage) for their training would decrease the cost, giving a greater optimal  $m$  and therefore faster development without any additional change, neither to the proposition nor to the proof.<sup>6</sup>

As long as there are no externalities of  $M$ , the total amount of managers, (5) is achieved for a constant value of  $m$  (since  $\frac{\partial c}{\partial m}$  is an increasing function), noted with  $\bar{m}$ . Therefore, every period each firm trains  $\bar{m}$  new managers and opens  $\bar{m}$  new production units, such that it has  $1 + \bar{m}$  production units. In the following period all these  $1 + \bar{m}$  managers are pre-trained, and every one of them heads a firm-unit which trains  $\bar{m}$  new managers and opens  $\bar{m}$  new production units, etc. The output per unit is not changing, and therefore  $\bar{m}$  is the growth rate of the technological sector.

Positive externalities of  $M$  means that along the development course, as  $M$  increases,  $\frac{\partial c(m, M_{t-1})}{\partial m}$  decreases, thus increasing of the optimal  $m$  along the process. Therefore, every period each firm open more production units than a firm in the period before, thus having an increasing growth rate of the technological sector over time.

QED

Note that since the labor market for trained managers is competitive and the existence, expansion and profitability of a firm depend on the existence of a manager,

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<sup>6</sup> Allowing potential managers to bid for an unbounded payment for training would lead to a higher payment in the initial period, thus allowing those managers to take advantage of their training for many periods, a payment bid which will decline in the following periods. This will lead to a decreasing  $m$  over time, similar to the one described under the second phase (sub-section 3.2). However, the process of catching up with existing technologies through on-the-job training still holds in this case.

firms actually pay the entire profit maximized above as a wage to the manager. This is also consistent with an interpretation of the manager being the entrepreneur or the owner of the firm. Note also, that since the managers and the workers in both sectors are the consumers in this economy, and since they consume their entire income, consumption equals production and a general equilibrium is achieved. Walras' Law allows for omitting the equilibrium analysis of this market.

For the explicit example one gets  $\frac{2m}{M_{t-1}^\beta} = A(\alpha A)^{\alpha/(1-\alpha)} - (\alpha A)^{1/(1-\alpha)}$ , and  $m$  is

calculated accordingly. When  $\beta = 0$  there are no externalities and  $m$  is constant over time, where  $\beta > 0$  means that  $m$  is indeed increasing along the process.

The growth rate of the whole economy is different from the growth rate of the technological sector, due to the output of the traditional sector. While on the initial periods a significant portion of the country's output is stemming from the traditional sector (and therefore the development of the technological sector may have only a minor effect over the growth rate of the entire output), this situation is reversed as the technological sector is expanding on the expense of the traditional one. Over time, the technological sector affects the overall growth rate more and more, up to the point where the traditional sector disappears and the whole development is stemming from the technological sector. As long as there are workers in the traditional sector (i.e.  $M_t \bar{l} \leq N$ ), the development process occurs as described above. However, from the period in which  $M_t \bar{l} > N$  firms will have to hire less than  $\bar{l}$ , and to pay them more than 1. This phase of development, the second phase, is analyzed below.

### 3.2 Second Phase

As the traditional sector disappears, firms are hiring fewer workers per production unit and paying them a higher wage. This reflects on the profitability of

the firm and accordingly on the number of production units (and managers) each firm is opening.

**Proposition 2:** second phase growth rate of the quantity of production units depends on the externalities in management training, such that the growth rate is diminishing without externalities, and may diminish, stay constant, or increase with externalities.

**Proof:** Since labor markets are competitive, each period, after the decision regarding  $m$  is made and  $M_t$  is set, each production unit will hire  $\frac{N - M_t}{M_t}$  workers, thus having

$\frac{N}{M_t}$  workers including the manager. Accordingly, the wage of a worker is  $f'\left(\frac{N}{M_t}\right)$ .

Let us note the quantity of workers including the manager with  $\tilde{l}$  and the wage with  $\tilde{w}$ , where  $\tilde{l}$  is decreasing in  $M$  and  $\tilde{w}$  is increasing.

$M_t = M_{t-1}(1 + E(m))$ , where  $E(m)$  is the average number of new units per firm. Since each firm is small, it can neglect its own effect over  $E(m)$  and therefore over  $M_t$  when deciding how many new units to open. Therefore, the firm's decision has no effect over  $\tilde{l}$  and  $\tilde{w}$ , and the maximization problem is:

$$(5) \quad \underset{m}{\text{Max}}\{(m+1)(f(\tilde{l}) - \tilde{w}(\tilde{l} - 1)) - \tilde{w}m - c(m, M_{t-1})\}$$

Accordingly:

$$\frac{\partial \pi(m)}{\partial m} = f(\tilde{l}) - \tilde{w}\tilde{l} - \frac{\partial c(m, M_{t-1})}{\partial m} = 0 \text{ and } m \text{ is set such that to}$$

$$(6) \quad \frac{\partial c(m, M_{t-1})}{\partial m} = f(\tilde{l}) - \tilde{w}\tilde{l}.$$

Each firm equates the marginal cost of training an additional manager to the marginal benefit. As development proceeds, marginal benefit is decreasing over time, each production unit hires fewer workers and pays a higher wage to each worker. Since

$\frac{\partial c(m, M_{t-1})}{\partial m}$  is increasing in  $m$ , as long as there are no externalities of  $M$ ,  $m$  is decreasing over time. Development pace obviously slows down. Externalities of  $M$  would decrease the left-hand side of (6) and therefore the optimal  $m$  may decrease, stay constant or increase over time.

Note, however, that the growth rate of the economy is lower than  $m$ , since every period there are less employees and lower production per unit. Note also that as before, the economy is in general equilibrium every period. Pre-trained managers extract all the profit as their wage, consumers are the managers and workers and consumers consume their entire income, therefore consumption equals the output.

QED

Let us follow the explicit example from before. Employment per unit is

$\tilde{l} = \frac{N}{M_t}$ , wage equals  $\tilde{w} = \alpha A \left( \frac{N}{M_t} \right)^{\alpha-1}$  and  $M_t = M_{t-1}(1+m)$ , and therefore equation

$$(6) \quad \text{translates} \quad \text{into} \quad \frac{2m}{M_{t-1}^\beta} = A \left( \frac{N}{M_{t-1}(1+m)} \right)^\alpha - \alpha A \left( \frac{N}{M_{t-1}(1+m)} \right)^\alpha, \quad \text{or}$$

$m(1+m)^\alpha = M_{t-1}^{\beta-\alpha} AN^\alpha (1-\alpha)/2$ . Since the left side is monotonically increasing in  $m$ ,

the change in  $m$  along the growth process depends on the change in  $M_{t-1}^{\beta-\alpha}$ . Weak or

no externalities mean that  $M_{t-1}^{\beta-\alpha}$  and  $m$  are decreasing over time, while strong

externalities ( $\beta > \alpha$ ) allow for an increasing  $m$ . The growth rate of that economy is

$$\frac{M_t f(\tilde{l}_t)}{M_{t-1} f(\tilde{l}_{t-1})} - 1, \text{ which translates into } (1+m)^{1-\alpha} - 1, \text{ and therefore is dependent on } m.$$

Growth in this model ceases when  $N=M$ . It may slow down gradually or may cease in a big-bang, depending on the externalities of  $M$  (and therefore on the behavior of  $m$ ). Deviating from the assumption that  $c(0, M_{t-1}) = 0$  may cause growth

to cease earlier, when  $M < N$ . Although this seems like an unfavorable result (which is common to some discrete growth models), it has a very favorable interpretation here. The cease of this type of growth requires a new type of growth – researching (or utilizing) new knowledge. The country has created the full capacity for absorbing the existing knowledge, such that it is utilizing the production function  $f(l)$  to its full potential thus exhausting this source for growth, completing the phase of development and catch-up, and future growth will depend on having a better technology.

Figure 1 presents the evolution of output and the amount of managers throughout the two phases of development. Without externalities, the quantity of managers initially grows at a constant rate, which slows down at the second phase until the entire population is trained. Output grows initially at a very slow pace (due to the substantial impact of the traditional sector), but once the quantity of managers is large enough, output growth speeds up. Eventually, once all the population is trained, growth ceases. When there are positive externalities (Managers2 in Figure 1), the pace of training speeds up and all the population is trained earlier. Output in this case grows faster as well (not shown). Figure 2 presents the composition of output throughout the process (without externalities). Note the substantial weight of the traditional sector in the first periods, which corresponds to the initially negligible growth rate of the total output in Figure 1.

**Figure 1 here**

**Figure 2 here**

### **3.3 Educational Choice and Heterogeneous Ability**

Human capital literature is dealing mostly with schooling as a form of acquiring human capital. Although this paper regards on-the-job training as the gateway to utilizing technology, schooling may very well be a pre-requisite for

training. Thus, firms may agree or be able to train only educated workers whereas the cost of education is incurred by the worker. Firms' decisions regarding the optimal  $m$  are not affected by the education, although the actual  $m$  is capped by the availability of educated workers. Assuming that the cost of education is low enough, the development process remains unaltered. Cost which is higher may lead to an earlier halt of the development.

Every period additional workers acquire education, such that the cost of acquiring education equals the future benefits from being a manager, adjusted for the discount factor and the probability of being trained in each of the subsequent periods. If the cost of education is low enough, all the workers are acquiring education such that there are no disturbances in the evolution of  $m$ . A higher cost may cause workers to choose not to acquire education starting from a certain period (note that being trained in latter periods yields lower returns from being a manager) thus stopping the development in an earlier phase.<sup>7</sup>

A common assumption in the human capital literature and income inequality literature is heterogeneity of ability within the population. One can easily add it to this model assuming that more skilled managers have a higher productivity in the unit they are heading. Obviously, firms would always prefer to train the most skilled among the existing workers. Moreover, a firm would train more managers per period in the initial periods (when the newly-trained manager is more skilled and therefore more profitable for the firm), and the optimal  $m$  would decrease over time.

### **3.4 Firm Specific Skills**

Training as modeled so far was general training, since managers could have left the training firms following the period of training enjoying the same productivity

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<sup>7</sup> A cost which is high enough may even prevent training beyond the first manager.

elsewhere. Introducing firm specific skills can be made in several ways and under various assumptions, generally speaking leaving the form of development and catch-up described above unaltered.

Let us stick to the previous assumption of one homogenous good. Introducing an assumption that training has firm specific characteristics, i.e. the productivity of a manager outside the firm is lower than productivity within the firm, combined with the fact that marginal productivity of a manager is constant, yields that the training firm can pay the manager more than his outside option of establishing a new firm. Therefore, the first firm created (the primary firm) would hire all the managers it trains, all the manager those managers train, etc., leaving the economy with one gigantic firm. Note, however, that development proceeds just as before, but with one large firm rather than many small ones. This situation calls for the introduction of one of the following two elements to the model: (1) Span of control problems – assuming that size also has disadvantages, such that the productivity of managers declines with size, yields that starting from a certain size manager's productivity inside the firm and outside the firm equate. From this point onwards managers would leave the primary firm to establish new ones, where every new firm may continue growing similarly to the primary firm. Therefore, one would get a distribution of firm sizes based on their time of establishment. (2) Diffusion of skills – assuming that firm specific skills diffuse over time and turn into general skills after a few periods would create a similar effect to the one above, and in addition would create a differentiate wage to managers. Managers whose specific skills have diffused would get a higher wage, and may establish new firms that will continue to grow.

A natural addition to firm specific skills is monopolistic competition with differentiated goods, where every firm produces a different good. As this is a model

of development and catch-up, local firms do not invent new goods, but rather imitate goods which already exist in the world. A small open economy, in which the prices of the goods are fixed, would behave similarly to the described above, with the primary firm being able to pay a higher wage to managers, and therefore only one gigantic firm would exist. However, any local nature of the differentiated goods may change this situation, preventing the existence of a single gigantic firm. Local characteristics of the goods, such as transportation costs, with a closed economy as an extreme case, will cause the relative price of a specific good to decrease with an increase in its local production, thus affecting the profitability of the firm. As the primary firm grows the price of its good decreases, creating a counter-effect to the firm specific skills and eventually, for a firm which is large enough, equating the marginal productivity outside the firm to the marginal productivity inside, causing managers to leave the firm. Therefore, one would get either a distribution of firm sizes based on their date of establishment or many small firms, depending on the effect of the changing prices compared to the effect of the firm specific skills. Introducing span of control problems or diffusion of skills instead of the local nature of the goods would prevent the creation of a single gigantic firm as well, similarly to their effect in the case of homogeneous goods.

Amid the homogenous goods and the differentiated goods, an intermediate method of modeling specific skills is having few technological sectors with sector specific skills in each one of them, such that managers are more productive in the sector in which they were trained. This would lead to the development of a gigantic primary sector, which evolves similarly to the development of the whole technological sector in sub-sections 3.1 and 3.2 (or the primary firm above), while the other sectors would not exist in that economy. Introducing diffusion of skills, closed

economy or other local characteristics of the goods (such that the relative prices of the goods produced in each sector are changing) or any other disadvantage to sector's size would prevent the existence of a single gigantic sector. It may, however, give rise to patterns of specialization, such as a situation of one or more large sectors, some small ones and some which may not exist at all in a certain economy, as is often found in small economies.

Note that all the aforementioned adjustments keep the general form of the process of development and catch up with existing technology through on-the-job training described in the original model.

#### **4. Evolvement of Income Inequality**

Throughout the development process analyzed on section 3 there are changes in the distribution of wages. The economy starts in a situation of complete equality. During the first phase the wage of trained managers is higher than that of the workers and the wages do not change (managers' wage actually increases when there are positive externalities), however the quantity of managers increases while that of the workers and traditional sector decreases. During the second phase there is an increase in the workers' wage and a decrease in managers' wage (the latter is relevant for externalities which are small enough), in addition to the growth in managers' share out of the population, leading eventually to complete equality again, when all the population is trained.

**Proposition 3:** For a population which is large enough and externalities which are not too large, the model exhibits an increase in inequality<sup>8</sup> in the first periods and a decline in inequality in the last periods.

**Proof:** Starting with complete equality, the introduction of managers with higher wage is increasing inequality. During initial periods of the first phase (described above) managers' wage is either constant or increasing, based on the externalities. Their initial share in the population is 0 and is increasing over time. Therefore, income inequality is increasing in the initial periods: (1) as the top quintile is populated by more and more managers the ratio of the upper quintile income to the lower quintile income is increasing; (2) Gini Index is increasing since a small group, which is increasing in size, is getting a higher wage than the rest of the population. Therefore inequality increases over the initial periods; note, however, that this process may not occur throughout the entire first phase, since inequality may start declining when managers become the majority of the population.

During the latter periods, income inequality changes from a positive level to complete equality again. The relative wage of managers decreases, since training managers is becoming less profitable and therefore each manager is heading fewer production units, and each unit is less profitable due to higher wage paid to workers, both leading to lower wage to the pre-trained manager.<sup>9</sup> The decline in managers' relative wage combined with the fact that managers become the majority of the population decreases income inequality: (1) the decline in managers' relative income decreases the ratio of the upper quintile income, populated by managers in latter periods, to the lower quintile income, populated at least by some workers. Moreover,

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<sup>8</sup> Using standard measurements such as the Gini Index or the ratio of the wage of the upper quintile to that of the lower.

<sup>9</sup> Positive externalities which are not too large may increase  $m$  over time but in a pace which does not compensate for the increase in worker's wage and the accompanying decline in production units' profits, thus causing pre-trained managers' relative wage to decline.

due to the growth in the quantity of managers the lower quintile is becoming populated by managers as well, thus reducing the ratio even more, eventually to complete equality between them; (2) Gini Index is declining since workers, which are a small group in the latter phases, are becoming scarce and their relative wage is catching up with the rest of the population. Therefore, income inequality increases in the first periods and declines in the last periods.

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Note that when externalities are large enough, managers' wages may increase in the last periods. Yet, as the development process ceases the economy is in complete equality, since everyone is a trained manager. Note also that assuming some positive externalities ( $0 < \beta < \alpha$ ) the model generates an increase in the relative wage of managers in the initial periods, and a decline in their relative wage in the latter periods (occurs for no externalities as well), a phenomena often traced throughout the evolvement of income inequality.

Although the proof characterizes an upward sloping income inequality curve in the initial periods and a downward sloping curve in the latter periods, it does not necessarily characterizes an inverted U-shaped curve, as a uniqueness of a critical point of the inequality index was not proved. This uniqueness, which may depend on exact specification, can easily be proved for some functional forms and some indexes. For example, for the case of no externalities and production and cost functions which yields  $m < 5$ , the first phase of development is coming to an end only after the top quintile is populated with only managers, therefore inequality measured as the ratio of top to lower quintile is increasing (as managers' share in the top quintile is increasing) or constant (from the period in which the top quintile is populated with only managers) throughout this phase. Throughout the second phase inequality is

decreasing since managers' wage is decreasing, thus decreasing top quintile's income, while workers wage is increasing and managers' share in lower quintile is increasing (up to the point in which all the population is trained), thus increasing the lower quintile income.

Figure 3 presents the Gini Index as a function of the output throughout the development process using the specific functional forms used as an example in section 3, with  $N = 1,000,000$ ,  $A = 2$ ,  $\alpha = 0.8$  and  $\beta = 0$ . Figure 4 presents the evolution of the Gini Index over time throughout the development process for two different sets of parameters of the production function. Both figures exhibit the inverted U-shaped evolution of inequality associated with Kuznets (1955) and often found in the relevant empirical literature. Note, however, that both the pace of evolution of income inequality and its magnitude depend on the specific parameters. The evolution of income inequality in this model also complies with Kuznets hypothesis that urbanization is associated with the upward sloping side of the curve, since the technological sector in this model, which is causing the rise in inequality in the initial periods, tends to concentrate in cities, thus attracting there both managers and workers.

**Figure 3 here**

**Figure 4 here**

## **5. Discussion**

This paper developed a highly stylized general equilibrium endogenous development-growth model in which firms are training managers to head new production units, thus expending production and increasing profitability. The ability of those managers to train new managers in the subsequent periods creates a process

of development and catch up with existing technology. Since both managers and firms are scarce resources in less developed countries, the model is suitable for the development analysis of these countries, unlike many other models. Being highly stylized, it may be adjusted to address many other aspects of development.

The analysis stresses two problems associated with development. The first is the accumulation of human capital, but not due to a schooling decision of the individuals, but rather due to a shortage in supply of training. The limited amount of advanced firms prevents large scale training and therefore prevents development.

The second problem is the dependence of development on the ability and incentives of the existing firms. When firms do not have incentives to grow, due to weak property rights or high tax rates for example, development would be slower or would even cease. When firms lack the ability to train, for example when current managers are unable to monitor newly trained managers, the same problem would occur. Looking at the training cost function described above can easily demonstrate it – when  $c$  is greater fewer managers would be trained, and when  $c(0)$  is positive and high enough, training may not occur. Therefore, the environment and nature of existing firms, not only the training potential (ability and education) of the existing population, matter for development.

The aforementioned development problems have implications for the analysis of various aid policies. Policies that increase the productivity of the traditional sector, a sector which can not grow by training managers due to the nature of its production function, may increase the output but would slow long run development, as it would decrease the profitability and the training incentive of the technological sector. Financing small businesses, as is often done using micro-credit, may help the founder but would not help overall development in case the business is in the traditional sector

or if the founder is unable to expand and train; accompanying the finance with proper guidance may be in place. Supporting the education system may not be enough if there are no on-the-job training opportunities in the economy. It will not improve the productivity, and may even be unworthy for the learner. Development policy should, therefore, foster and encourage expansion and training, and should be careful with other tools. Note also the slow evolution of output in the model in the initial periods, in which the traditional sector is the most significant one, and the increase in pace as the quantity of managers and the weight of the technological sector in the economic activity increase. Proper policy, therefore, may have an effect over the technological sector, but a noticeable effect over total output may take time.

This paper has shed new light on the problems of development of the poorest among the less developed countries. Hopefully, it will be a building block in improving the economic performance of these countries.

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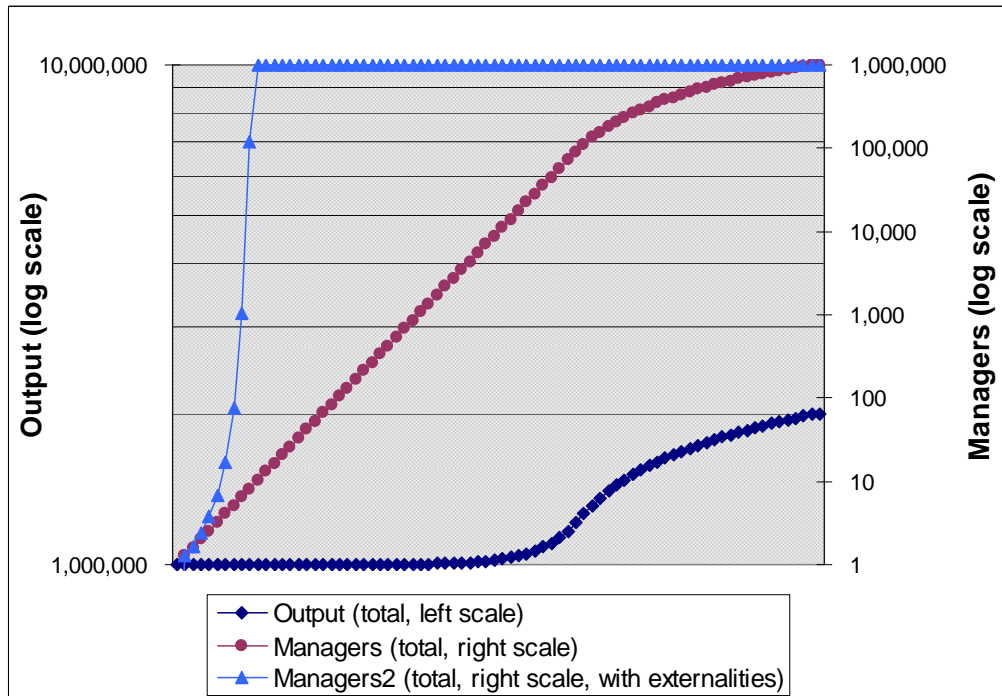
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**Figure 1 – Output and Managers throughout the Development Process**

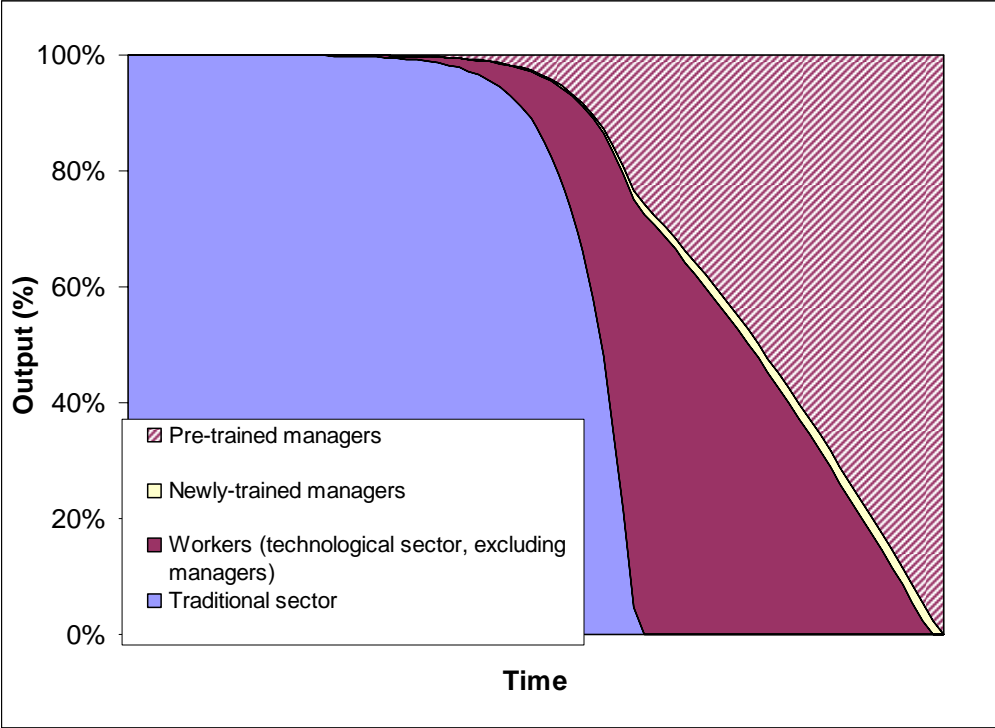
Plotted for  $N=1,000,000$  ,  $f(l) = 2l^{0.8}$  and  $c(m, M_{t-1}) = m^2$

Managers2 is plotted for  $N=1,000,000$  ,  $f(l) = 2l^{0.8}$  and  $c(m, M_{t-1}) = \frac{m^2}{M_{t-1}^{0.9}}$



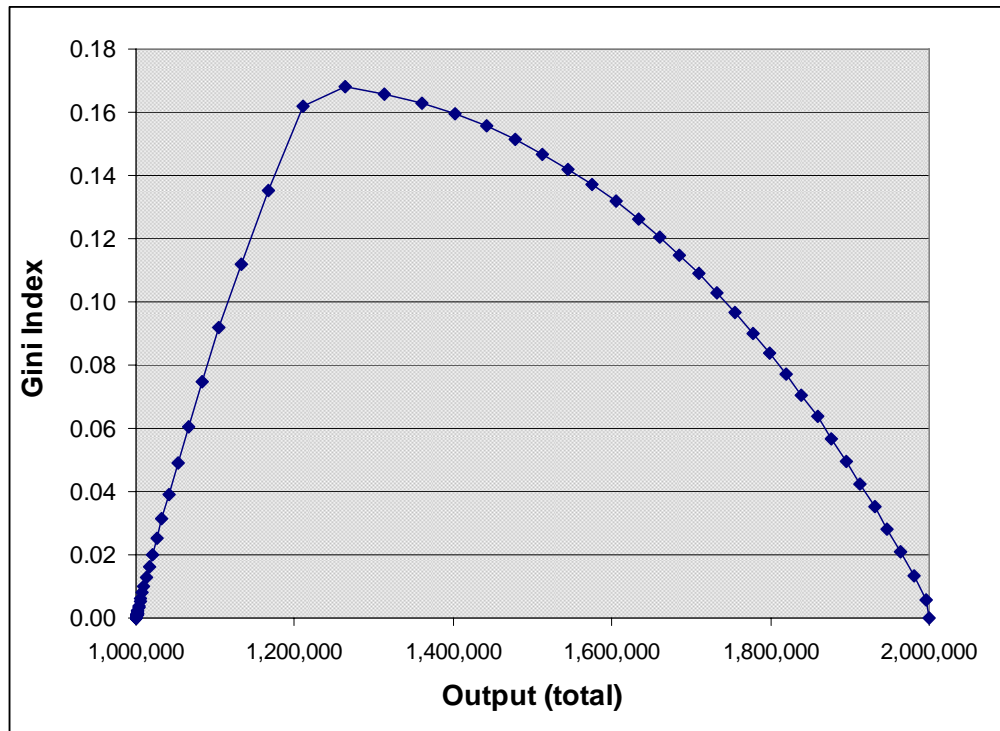
**Figure 2 – Compositions of Output throughout the Development Process**

Plotted for  $N=1,000,000$  ,  $f(l) = 2l^{0.8}$  and  $c(m, M_{t-1}) = m^2$



**Figure 3 – Income Inequality (Gini Index) throughout the Development Process**

Plotted for  $N=1,000,000$  ,  $f(l) = 2l^{0.8}$  and  $c(m, M_{t-1}) = m^2$



**Figure 4 – Income Inequality (Gini Index) throughout the Development Process**

GINI1 - Plotted for  $N=1,000,000$  ,  $f(l) = 2l^{0.8}$  and  $c(m, M_{t-1}) = m^2$

GINI2 - Plotted for  $N=1,000,000$  ,  $f(l) = 3l^{0.6}$  and  $c(m, M_{t-1}) = m^2$

