PHASE SPACE RECONSTRUCTION AND NONLINEAR EQUILIBRIUM DYNAMICS APPLIED TO U.S. MEAT DEMAND

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Abstract: We investigate dynamic interactions of market driven consumer behavior using phase space reconstruction, which has been developed to analyze nonlinear dynamical systems. This approach provides important and unique empirical insights into U.S. beef demand. Results from the phase space reconstruction analysis demonstrate distinct differences between intertemporal shorter run impacts from food safety outbreaks (e.g., E. Coli) relative to longer run health effects (e.g., cholesterol). Demand adjustments due to cholesterol reflect permanent changes, while consumers react to food safety scares by adjusting consumption for a short period of time and then return to their normal seasonal patterns and levels of consumption.

Key Words: nonlinear time series, phase space reconstruction, food safety, health effects

JEL Classification:

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Introduction

Recent literature has been concerned with the impacts of health and food safety events on beef demand. Health events are defined as those events dealing with the dissemination of information concerning attributes of a good that have an effect on consumer health. One such event took place in the late eighties when information was published on the link between heart disease and cholesterol in red meat. Food safety events are those that pertain to contaminate outbreaks in a particular consumption good. The 1993 E. coli outbreak in the beef resulted in severe illness and several deaths in the United States.

Generally, health events have been speculated to last much longer than food safety events. Kinnucan (1997) estimated the presence of a structural change in beef consumption due to the release of cholesterol related information. Indeed, cholesterol and heart disease are still important concerns. On the other hand, E. coli has been shown to have fairly short and small consumer responses, (Piggott and Marsh, 2004; Mazzocchi, 2006). Even though E. coli may still contaminate beef, the contaminant does not have a latent effect as it is usually cooked out before consumption. The responses that consumers have had to E. coli have also been compared to the consumer responses to BSE. In particular, the 2003 BSE outbreak has been extensively studied and little evidence has been found that differentiates the impact of E. coli and BSE (Piggott and Marsh, 2004; Resende-Filho and Buhr, 2007). These studies have been limited in scope due to the restrictions of static econometric modeling; therefore a proper overall delineation of the dynamic consumer reactions to health events and different food safety events has yet to be performed. This paper demonstrates that health events result in permanent consumer behavior changes, and food safety events result in short deviations from normal consumer behavior. In addition we show that the differing food safety events result in differing consumer responses.

Determining how these events differ in how they affect consumption allows for a more effective response by both suppliers. For instance, during the 2003 BSE outbreak beef wholesalers were able to anticipate that consumption would only decrease slightly. They then sourced domestic beef at very low prices that had been driven down as foreign countries banned U.S. exports increasing domestic supply. Companies such as McDonald's that responded in this manner experienced large profits while those that did not share in the foresight suffered.

Recent papers have attempted to investigate the impacts of health (e.g., Kinnucan et al., 1997) and food safety events (e.g., Piggott and Marsh, 2004; Mazzocchi, 2006) using a wide variety of econometric tools. Each study has provided important empirical results by extracting measures of consumer behavior from time series data using standard econometric techniques. However, the standard techniques have been limited in their capability to identify the differences between particular food safety and health events. Zhen (2005) conceptualized a model based upon Becker's theory of rational addiction, providing evidence that beef consumption in the United States is persistent and that consumers may be grouped by their persistence. He argued that the more myopic the consumer the less responsive they are to food scares and health effects. However, his a priori assumption that health effects and demographic variables result in a continuous decrease in consumption may be constraining the model to underestimate later food scare reactions. Mazzocchi created a dynamic almost ideal demand system to determine the time-varying reactions consumers have to outbreaks. He showed that the inclusion of autoregressive parameters as consumer reactions provided decent short term forecast ability when estimating the impact of food scares (Mazzocchi, 2006). If, however, the food scare or health effect has a longer run impact the dynamic almost ideal demand system will be unable to capture it. Piggott and Marsh used a general almost ideal demand system to examine the impacts of public food safety information on US meat demand. They found the 1993 E. coli and 2003 BSE food safety concerns to have small and short-lived, but statistically significant impacts on meat demand. However, the impacts of these different food safety events could not be distinguished. Applying AIDS models to the beef industry provides useful information but requires a substantial amount of constraining assumptions.[1] In addition, this previous empirical evidence has suggested that impacts from food safety outbreaks are short run and structurally different from health effects that tend to be long run. Inherently, the theoretical basis for these approaches has been static in nature that is then estimated by an empirical model augment with dynamic components, which may or may not capture the dynamic impacts present in beef consumption.

In response to such pitfalls, novel techniques that are inherently dynamic provide alternative means to examine time series data. Phase space reconstruction is one such technique that has been developed to analyze nonlinear dynamical systems. Phase space reconstruction, using nonlinear time series techniques, allows for the extraction of an underlying structural system from a single observed time series of data that can clearly delineate qualitative dynamic impacts of health or food safety impacts on consumer demand. It is an innovative empirical tool that provides unique insights into behavioral

[1] General AIDS models such as these are very useful in determining short term adjustments in behavior, as both Mazzocchi and Piggott and Marsh did, but the models are fundamentally restricted to this type of analysis. They have difficulty delineating long run dynamics from short run dynamics.

characteristics.[2] With more insight into consumer behavioral reactions a more accurate model may be built.

Indeed, our results from a phase space reconstruction analysis demonstrate distinct differences among the intertemporal responses. Through phase space reconstruction we are able to delineate the effects of different types of contaminate outbreaks on beef consumption. We show that the nonlinear dynamics present in American beef consumption have remained somewhat similar over the last twenty years with the exception of the health effect of cholesterol.

Since its development phase space reconstruction has become an essential part of nonlinear dynamics (Packard et al., 1980, Takens, 1981). It has been incorporated into various areas of research from Schaffer and Kot's SIER model of epidemics to Zaldívar's forecasting of Venice water levels, phase space reconstruction is the qualitative benchmark for nonlinear analysis (Schaffer and Kot, 1985, Zaldívar et al., 2000). It allows for the basic properties of the system to be determined and subsequent qualitative analysis to be performed without any prior knowledge of a system. This is analogous to nonparametric regression, which allows for the relationship of variables to be determined without imposing restrictions or prior functional form. Indeed, phase space reconstruction could be thought of as a nonparametric approach to nonlinear time series modeling.

By incorporating phase space reconstruction into the analysis of beef consumption we have provided the economic discipline with not only a better understanding of consumer behavior in general but with a new tool to be used in all types of future

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[2] Phase space reconstruction does not necessarily lead to a final definitive model in-of-itself, but rather can provide information to specify a more complete structural model.

research.[3] The techniques involved with phase space reconstruction, discussed below in the Theoretical Framework section, have been improved upon making them asymptotically efficient and more easily understood for those wishing to use the technique.

The study proceeds in the following manner; an overview of nonlinear time series methods is presented followed by the theory and method for reconstructing phase space in the Theoretical Framework. The applied phase space reconstruction of the United States beef consumption follows in Consumer Beef Demand with results and discussion. Consumer reactions to health effects and food safety are then delineated and compared to existing research with concluding remarks for future modeling purposes.

Theoretical Framework

The nonlinear time series methods discussed in this paper are motivated and based on the theory of dynamical systems (Takens, 1981). The general idea is that a single scalar time series may have sufficient information with which to reconstruct a dynamical system, much like a single stain of DNA contains sufficient information to reproduce an entire organism. As such, time evolution is defined in a phase space.[4] Dynamical systems are usually defined by a set of first-order ordinary differential equations and have been shown to provide valuable insight into market dynamics (Chavas and Holt, 1993). The mathematical theory of ordinary differential equations ensures the existence and

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[3] Since its development, phase space reconstruction has been widely used as a tool to detect chaos. We make no assumptions as to the chaotic behavior of consumption nor are we trying to test for it. Rather, the primary goal of using phase space reconstruction in this analysis is to better understand the qualitative dynamics of consumer behavior.

[4] Typically phase space is defined as the space in which some geometric structure exists. In very general terms every trajectory of the structure of question may be represented as a coordinate in its particular phase space. For qualitative analysis we will always be referring to phase spaces of two and/or three dimensions.

uniqueness of the trajectories, if certain conditions are met (Packard et al., 1980, Shone, 2002).

Data are often observed as a temporal sequence of scalar values. For any event, n outcomes are observed as a subset of the total population and are denoted by the time series vector $X_t = [x(t), x(t-1), ..., x(t-n)]'$. For future reference the τ^{th} lag of this vector will be referred to as $X_{t-\tau} = [x(t-\tau), x(t-1-\tau), ..., x(t-n-\tau)]'$. The challenge is to convert the observations into state vectors and reproduce dynamics in phase space. This is the essence of phase space reconstruction, which we solve using the method of delays discussed ahead. Phase space reconstruction is a diffeomorphism that reproduces a time series on a plane that mirrors the phase portrait of the underlying system.[5] This reproduction makes it possible to qualitatively delineate short or long run behavioral processes that evolve over time and generally better understand the qualitative nature of the dynamical system.

Embedding

The idea of embedding attractors onto different spaces and in different dimensions is an important concept in the theory of dynamical systems.[6] It was not until Packard first proposed that this be done from measured time series that the idea of phase space reconstruction was formed (Packard et al., 1980). Packard proved that the embedding of the geometry of a strange attractor may be represented by a series of differential

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[5] A diffeomorphism is a smooth function, Φ, that maps one differential manifold, *M*, onto another, *N*, whose inverse, Φ', is also a smooth function that maps *N* onto *M*. This mapping preserves all geometric properties of the original figure. Much like a topographical map preserves all geometric properties of the earth.

[6] An embedding is the mapping process used to reproduce geometric figures onto different spaces. Again it is analogous to creating a two-dimensional map of the three-dimensional world. Not all embeddings are diffeomorphisms, just like not all maps contain all the properties of the area they cover. Nonetheless, even though road maps don't usually contain elevation gain they provide a great deal of information.

equations.[7] Takens extended this proof to encompass what is now known as the Method of Delays (Takens, 1981). The Method of Delays is a diffeomorphism of an attractor with dimension m onto a phase space of dimension n where $n \geq 2m+1$. For empirical application, the Method of Delays requires an optimal time lag τ be chosen followed by a minimum embedding dimension λ. Once the two parameters are estimated, the time series X_t will generate a reconstructed phase space matrix $Y_\lambda = \lfloor X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(\lambda-1)\tau} \rfloor$ with dimension $[(n-\lambda\tau)\times\lambda]$.[8]

The time lag τ is paramount to empirical applications of Takens' theorem. While the condition $n \geq 2m+1$ is sufficient, it is not necessary. By choosing a time lag that yields the highest independence between the column vectors in matrix Y_λ the geometry of the original manifold will be preserved even when the time series is contaminated with noise.

The time lag needs to be chosen optimally. If it is too small the approximation will be smooth but there will exist a high degree of correlation between components. This has the potential to force the trajectories of the attractor to lie on the diagonal in the embedding space (Broomhead and King, 1986). If, on the other hand, the time lag is chosen to be too large the dynamics of the system may unfold between components and therefore be unobserved. The optimal time lag is that which preserves the largest amount of information between components while achieving the largest degree of independence.

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[7] An attractor is a subset of a space onto which a system evolves to over time. A strange attractor is an attractor that allows for a greater degree of flexibility in that the subset of the space may be fractal, i.e., the dimension of the space does not have to be a real integer.

[8] As discussed before, the process of phase space reconstruction is much like map making. The reconstruction is a map containing all geometric properties of the original system that drives the dynamics of the observed time series. Through Takens' embedding theorem it is possible to extract this map of the underlying dynamics from a single time series.

Time Lag for Embedding - Mutual Information

The mutual information coefficient was developed as an entropy measure of global dependencies between two random variables (Fraser and Swinney, 1986). The dependencies measured are both linear and nonlinear making it an ideal candidate for choosing an optimal time lag. In estimating the optimal time lag we are essentially asking the question: How dependent is X_t on $X_{t-\tau}$? To answer this question Fraser and Swinney defined dependence based upon conditional entropies and called it the mutual information function[9]

$$I(X_t, X_{t-\tau}) = H(X_t | X_{t-\tau}) = H(X_{t-\tau}, X_t) - H(X_{t-\tau})$$

where $H(X_t)$ is Shannon's entropy

$$H(X_t) = -\sum_t P_{x_t}(x(t)) \log P_{x_t}(x(t))$$

and

$$H(X_{t-\tau}, X_t) = -\sum_{t,t-\tau} P_{x_t x_{t-\tau}}(x(t), x(t-\tau)) \log\left[P_{x_t x_{t-\tau}}(x(t), x(t-\tau))\right]$$

(Shannon and Weaver, 1949).

The Mutual Information Function is defined as the combination of joint and marginal probabilities of the outcomes from an event in a sequence while increasing the time lag τ between components:

$$I(X_t, X_{t-\tau}) = \sum_{n-\tau} \sum_{n-\tau} P(X_t, X_{t-\tau}) \log\left[P(X_t, X_{t-\tau}) / P(X_t) P(X_{t-\tau})\right].$$

[9] When phase space reconstruction was first developed the autocorrelation function was used to find the optimal time lag. This measure of dependency is severely inferior for nonlinear analysis as it is strictly a measure of linear relation. Furthermore, the autocorrelation function hinges on estimating the sample moments of a time series. The mutual information function makes no assumptions about moments or the underlying distributions of time series; it is strictly observation dependent and therefore void of all misspecification and assumption errors.

Choosing the time lag that yields the first local minimum of the mutual information function ensures independence of components with a maximum amount of new information (Fraser and Swinney, 1986).[10] The first minimum is chosen as the optimal time lag based on the optimality conditions defined above so that it is neither too small nor too large, ensuring that the attractor unfolds correctly.[11]

Estimating the mutual information function hinges on estimating the probability density function of a time series and its lagged values. This has traditionally been done using histogram estimators that are perceived as the "most straightforward and widespread approach" (Dionísio et al, 2006). The histogram method of estimating density functions uniformly weights observations within a predetermined window. If the time series contains a large portion of observations located close together and some that are spread out, the histogram method will inconsistently estimate the probability density function. Algorithms have been developed that vary the window size based upon how close observations are located to each other but they are computationally intense and not easily programmed (Fraser and Swinney, 1986).

As an alternative approach we apply nonparametric estimation using kernel density approximations as a method for estimating the mutual information function.[12] In

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[10] There have been many entropy measures of dependence developed over the years in economics, see Granger et al. (2004), and Maasoumi and Racine (2002). The main points stressed are usually to the extent which measures are a metric, so that comparisons may be made, and which are just measures of divergence. Phase space reconstruction only requires the independence of its coordinate vectors, which may be measured as either distance or divergence. It is out of the scope of this paper to compare measures of dependence so the mutual information function, with improvements, is used, as it is the norm in phase space reconstruction literature.

[11] If the global minimum of the mutual information function were used as the optimal time lag the potential would be for the nonlinear system to have already completed a full cycle so that the estimate would include redundancy of the system as well. This could force the phase space reconstruction to no longer be monotonic, thus enveloping dynamical structure.

[12] Mittelhammer et al 2000 provide detailed information on nonparametric estimation and GAUSS coding examples. Simulations comparing the nonparametric and histogram methods (not reported here) demonstrated improved performance of nonparametric methods over the histogram approach in smaller

every instance the nonparametric approach took substantially less computational time. In addition to being less computationally burdensome, under appropriate conditions, the nonparametric method of estimating the mutual information function is also asymptotically efficient. By using kernel weights the possible inefficiencies encountered with the histogram method of estimating the mutual information function are minimized.

Embedding Dimension - False Nearest Neighbors

Given the choice of optimal time lag, the minimum embedding dimension λ can be estimated. Kennel and Brown (1992) developed the False Nearest Neighbors technique (discussed below) for choosing a minimum embedding dimension. Aittokallio (1999) suggested the embedding dimension must be chosen properly or the reconstruction may not reflect the original manifold. If λ is too small the reconstruction cannot unfold the geometry of the possible strange attractor.[13] If λ is too large procedures used to determine basic properties of the system and qualitative analysis may become unreliable (Aittokallio, et al., 1999, Kennel, et al., 1992).

The False Nearest Neighbors technique uses Euclidean distances to determine if the vectors of Y_λ are still "close" as the dimension of the phase space is increased. By calculating the Euclidean distance between Y_λ vectors before and after an increase in dimension, it is possible to determine if the vectors are actual nearest neighbors or "false" nearest neighbors. The test statistic developed by Kennel and Brown defining neighbors to be false is: $\frac{x(t+d)-x(n(t)+d)}{\|y_t - y_{n(t)}\|} > R_{tol}$ where $x(t+d)$ denotes the last coordinate in the t^{th} row

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sample situations. Improved performance measures included both in accuracy and precision for estimates of the time lag parameter and in computational time. Both methods converged to one another as the sample size increased as anticipated.

[13] An example of an embedding dimension being too small would be a 2-dimensional representation of a cube. In 2-dimensional space the cube appears to be a square. In 3-dimensional space the true geometry of the cube is clearly not a square but a much more complex figure.

of the phase space reconstruction matrix $Y_{\lambda+1}$, $n(t)$ denotes the nearest neighbor in Euclidean distance of t for each row vector y_t in matrix Y_λ, and R_{tol} is the desired tolerance level. When the percentage of false nearest neighbors is minimized or drops below a preset threshold for the entire system, the minimum embedding dimension for phase space reconstruction is found (Kennel, et al., 1992).

The graphical false nearest neighbor test works in similar fashion; the test plots the density of false nearest neighbors in time delay space. Using the same notation as above $\left|x(t+d) - x(n(t)+d)\right|$ is plotted on the y-axis against $\frac{\|y_t - y_{n(t)}\|}{\sqrt{d}}$ on the x-axis. The first embedding dimension which yields the entire density of points contained below a 90° line is the minimum embedding dimension (Aittokallio, et al., 1999).

Consumer Beef Demand

Beef is a well-known staple in the American diet, and has been extensively studied (Chavas, 2000, Kinnucan, et al., 1997, Mazzocchi, 2006, Patil, et al., 2005, Piggott and Marsh, 2004; Zhen, 2006; Zhen, 2005). United States consumers eat roughly as much beef as they did forty years ago with consumption peaking in the 1970s. See Figure 1 for the time series graph of quarterly beef consumption per capita from 1960-2005. Table 1 reports descriptive statistics. Figure 1 illustrates the seasonal patterns and trends that have occurred throughout the history of beef consumption. Demand peaks in the summer months and is lowest in the winter. The average difference between the first and third quarter is 0.71, see Table 1. This difference increases slightly for the period after 1980 and then stays relatively consistent. In addition to the seasonal behavior, there appears to be an average level about which consumption has fluctuated since 1990.

We examine four events for three different quality concerns that stand out in the beef consumption literature. First is the reaction consumers had to the information regarding the negative health effects of cholesterol. The cholesterol health effect is commonly associated with the downward trend that consumption takes in the mid-eighties. The next two events can be attributed to food safety scares regarding E. coli outbreaks. The first E. coli outbreak took place in 1993 when several people became sick after consuming Jack-in-the-Box products. The second outbreak of E. coli occurred in the mid-west in 1997 and resulted in a large recall of beef. The last event is the result of BSE being detected in cattle in the Northwest in 2003. These events are also identified in Figure 1.

Empirical Issues

The empirical process progresses in several steps. First, the optimal lag is estimated and, second, the appropriate embedding dimension is determined. Third, the phase space reconstruction is completed and interpreted for U.S. beef demand. Fourth, further statistical tests are investigated to identify different health and food safety events.

The first minimum of the mutual information function determines the optimal time lag for the phase space reconstruction. The optimal time lag for beef consumption is estimated to be $\tau = 4$, as indicated in Table 2 where the mutual information function reaches its first local minimum. Interestingly, the autocorrelation function has a first minimum at lag two and is nonzero throughout the study period. At this time lag the mutual information function indicates that there is still some redundancy between the time series and its lagged vector. If used, the autocorrelation function would have resulted in a phase space reconstruction erroneously forced upon the 45° line. The difference in the mutual information and autocorrelation functions suggests evidence of nonlinear time series processes present in the data series.

The graphical false nearest neighbors test is implemented, shown in Figure 2, to determine the minimum embedding dimension for the phase space reconstruction. Clearly the entire density of observations is contained below a line of degree less than 90 for the two-dimension case making the minimum embedding dimension $\lambda = 2$. Using the two parameter estimates we can create a graphical representation of the underlying dynamics that drive beef demand from 1960 to 2005, see Figure 3.

Interpreting the phase space reconstruction becomes clearer by comparing the original demand series Figure 1 and the reconstructed phase space in Figure 3. In Figure 3, the horizontal axis is the observed time series and the vertical axis is the time series lagged four periods. Beginning in 1960, per capita consumption was under 16 pounds per person per quarter, which appears in the lower left hand corner of Figure 3. As per capita consumption increased and peaked in the mid to late 1970s, the trajectory moved to the upper righted hand portion of the phase space. As per capita consumption began declining, the reconstructed trajectory began transitioning back towards the lower left hand part of the phase space during the 1980s. By the early 1990s the consumption series stabilized to seasonal pattern just under 17 pounds per person per quarter, reflecting a persistent cycle in the lower left part of phase space.

Seasonality of beef consumption is exhibited in the original time series and the reconstruction; people consistently eat more beef in the summer than in the winter. The reason, traditional American winter dishes consist mostly of beef's biggest substitutes poultry and pork, turkey for Thanksgiving; duck and ham for the holidays, while the traditional American summer dish is primarily barbequed beef. What is apparent from the reconstruction is that the difference between winter and summer consumption has been relatively consistent over recent time, which is consistent with findings from previous studies.

The phase space reconstruction in Figure 3 shows the period from 1961-1980 was a time of transitions. In the early part of the series, the consumer increasingly incorporated beef as a part of their daily diet. Fast food restaurants such as McDonalds founded in the early fifties were beginning to takeoff. Then in the late seventies the price of beef began to increase dramatically. The consumer decreased the amount of beef she ate until a relatively stable (albeit brief) trajectory emerged in the early 1980s.

The main purpose of this application is to examine the subsequent period from 1980 to 2005, focusing on reported consumer reactions to the health concern cholesterol and food safety concerns Escherichia coli (E. coli) and bovine spongiform encephalopathy (BSE). Focusing the analysis on the period from 1980-2005 allows for an easier delineation of the health and food safety effects on beef consumption.

Consumer Reactions to Health Effects

During the late eighties information was published on the negative effects cholesterol in beef has on health. Evidence suggested that health effects resulted in a decrease in U.S. beef consumption (Kinnucan et al., 1997). Figure 4 shows a subspace of the reconstruction from 1980 to 2005. The reconstruction shows a period of consistent consumption in the early 1980s (the cyclical pattern in the upper right sector), a transition period (reflecting consumer reaction to cholesterol), and the set of more recent stable cycles at a lower level of consumption (in the lower left sector). Using a k-means clustering algorithm further delineates these patterns and transition periods.[14] The result of the cluster analysis places all the trajectory data prior to 1989 in one cluster and all the trajectory data after 1990 in another. The clustering results support the phase space reconstruction that the effects of negative information concerning health are statistically and economically significant. Consumers reacted to the information by decreasing their average level of consumption permanently while retaining a persistent seasonal pattern. The adjustment period illustrated in Figure 4 is consistent with a longer run behavioral response. This aligns with the empirical findings of Kinnucan et al., (1997).

Modeling consumer reactions to health effects requires proper behavioral responses to be delineated. Zhen's (2006) analysis of consumer response to contaminate outbreaks in beef, grouped demographic variables and the health effects of cholesterol together and modeled them as a downward trend to capture any deterministic behavior. This technique is widely used and accepted, however, a trend implies that the reaction consumers have had to cholesterol is constantly decreasing. The phase space reconstruction shows that consumption is not continually decreasing. Rather, an adjustment period occurred after which consumption returned to its regular trajectory at a permanently lower level. The two clusters in Figure 4 are almost identical in shape indicating that consumer behavior, post health effects, is almost identical to consumer behavior before the health effect information was released. This suggests a long run shift in consumption levels opposed to a change in behavior, as a trend adjustment would

[14] The algorithm chooses two clusters at random and then calculates the Euclidean distance from all to the cluster means. Each observation is then reclassified to the cluster that contains the mean it is closer to. The new clusters are defined and the process repeats until observations no longer change clusters (Johnson and Wichern, 2002).

imply. An incorrect specification of health effects in the 1980's with a downward trend in consumption will cause the underestimation of subsequent food safety effects.

Consumer Reactions to E.Coli and BSE

To examine consumer's reactions to more recent E. coli and BSE concerns the phase space trajectory is presented from 1990 to 2005. As can be seen in the phase space reconstruction, the reactions from E. coli and BSE outbreaks are different for health concerns and from one another. During an outbreak of E. coli or BSE, beef consumption was perturbed for a brief period of time until consumption returned to the normal cyclical pattern.

To delineate the outbreak trajectories from the remaining data a k-means clustering algorithm (as described above) was implemented on the phase space reconstruction. Figure 6 plots the phase space reconstruction highlighting the seasonal clusters. The resulting northwest cluster contains spring and summer quarter trajectory data and southeast cluster contains winter and fall trajectory data (with two spring observations in 1993 and 1994).

Next, the magnitude and duration of the outbreaks are examined using Euclidean distances from the respective cluster means and 99% normal confidence ellipsoids. Data from the individual clusters are comprehensively tested under the null hypothesis of being normally distributed. First, the two coordinate vectors of the phase space reconstruction, $x(t)$ and $x(t-4)$, are tested for normality independently using the Shapiro-Francia W' test for normal data (Royston, 1983). Testing the null hypothesis of normality versus the alternative non-normality, the Shapiro-Francia W' test null hypothesis cannot be rejected at the $\alpha = 0.05$ level in each case, see Table 3 for the test statistic and critical values. For the individual vectors testing normal, the next step is to test for joint normality. To do this we employ the methods described by Johnson and Wichern (2002). Constructing a Chi-Square plot of the squared generalized distances reveals that the joint probability is in fact bivariate normally distributed (the Chi-Square plot is analogous to the Normal Quantile Plot but for checking bivariate normality). Another check of bivariate normality is to check the percentage of observations that lie within the 50% confidence ellipsoid, $(x-\mu)'\Sigma^{-1}(x-\mu) \leq \chi_2^2(.5)$, see Tables 4 and 5. With the normal distribution assumption verified outlying observations are those defined as being outside the 99% confidence ellipsoid defined above; see Tables 4 and 5. The chi-square probability values reported in Tables 4 and 5 refer to the confidence ellipsoid that the trajectories lie on. Applying this outlier analysis to the phase space reconstruction makes it possible to differentiate the perturbation magnitude of E. coli and BSE events discussed ahead.

Figures 7, 8, and 9 show the phase space reconstruction of beef consumption when individual E. coli and BSE events are isolated from one another. For example, Figure 7 includes the phase space trajectory from 1990 to 2005 and the 1993 E. Coli event but not the 1997 E. Coli nor the 2003 BSE event. The reconstruction of the first outbreak of E. coli in 1993, Figure 7, is noticeably larger than the second E. coli outbreak of 1997, Figure 8. This is also evident in the results reported in Tables 3 and 4 where the 1993 (1997) E. coli trajectory has a maximum chi-square probability of 0.99 (0.83). The corresponding Euclidean distances from the spring/summer cluster mean are larger for 1993 than for 1997. The difference between the two outbreak trajectories can be attributed to the fact that consumers may have become less sensitive to E. coli (as E. coli can be cooked out of beef, which will decrease the impact of the outbreak if the consumer takes the proper measures).

The 2003 BSE incident, Figure 9, resulted in a greater perturbation of consumption than those due to E. coli. The Euclidean distance from the cluster mean is larger for the 2003 BSE trajectory than the 1993 E. coli trajectory. In addition, the 2003 BSE trajectory lies outside the 99.5% confidence ellipse of it cluster while the 1993 E. coli trajectory does not. Unlike E. coli, BSE cannot simple be cooked out of beef. The uncertain nature of BSE appears to play an important role in consumer reactions. There are no preventions for being contaminated by BSE other than abstaining from eating beef. In addition to the lack of preventative measures, some evidence has been found for a causal link between BSE and Creutzfeldt-Jakob disease. BSE is shown to be present in the central nervous system and bone marrow of cattle, portions not normally consumed, while E. coli may be found in meat. Interestingly, the risk of becoming contaminated by BSE is much lower than E. coli. This gives evidence that people's behavior is affected more by the latent hazard of a potential longer run health impact.

The difference between the E. coli and BSE events is an important and interesting result. Piggott and Marsh (2004) estimating a demand system of beef, pork, and poultry found no significant difference between non-domestic BSE information and E. coli public food safety information as measured by the number of newspaper articles. However, the BSE media index used by Piggott and Marsh (2004) did not include information from the 2003 BSE event in Canada or the US. Resende-Filho and Buhr (2007) modify the same model to identify the willingness to pay for a National Animal Identification System and are unclear as to the impact the 2003 BSE outbreak had on beef consumption. In all, the nonlinear time series approach suggests different behavioral responses due to the BSE and E. Coli events.

Concluding Remarks

The implementation of phase space reconstruction provides a novel approach to investigate dynamical systems that are pervasive in interesting and complex economic problems. It further opens a door for future research in nonlinear time series econometrics and provides important motivation by which to use empirical information to better specify theoretical models. Our analysis provides relevant, interesting, and unique insights into the consumer demand for beef and to the impacts of health and food safety on beef demand.

In addition to highlighting the usefulness of phase space reconstruction, we have contributed to its empirical foundation. The nonparametric method implemented to estimate the mutual information functions has been improved making it both more efficient and computationally simple. The mutual information coefficient is not only useful in phase space reconstruction. It is being used in place of the correlation coefficient in a variety of analysis. Kraskov et al., (2005) use it as a coefficient to base a hierarchical clustering algorithm with a high degree of accuracy. Being that it is both a linear and nonlinear global determinant of dependency between two random variables, its application is sure to be wide spread.

The recent empirical evidence suggesting a different consumer response to health effects and food safety is evident in the phase space reconstruction. The long run health effect of cholesterol has caused consumers to shift their consumption behavior to a lower level while retaining a persistent seasonal pattern. This lower level contains some of the same behavioral dynamics present before the health information was released.

The effects of food safety information in the phase space reconstruction are shown as temporary adjustments from the consumer's phase space trajectory. These adjustments appear to be dependent on the particular contaminate. Consumers are apparently learning to better prepare food so that the impact of E. coli outbreaks have lessened over time. The evidence regarding the dramatic reaction to the BSE outbreak shows that consumers might be affected more by the latent hazard of a potential longer run health impact. For a predetermined consumption good such beef, outbreaks of contaminates have short run negative impacts. Consumers return to a steady trajectory when they believe the risk of contamination has decreased to a significant level.

In addition to the insights on beef consumption this analysis provides a framework for better understanding consumer behavior in general. As shown, if the consumption good, contaminate, or factor that affects perceived quality is relatively new to a society they will react in a much more severe manner. This suggests the need for further research into the effects that increased consumer knowledge pre-outbreak has on decreasing the impact of contaminates during an outbreak.

Tables

Descriptive Statistics	
Mean	18.62
Standard Deviation	2.14
Minimum	14.92
Maximum	24.38
Mean post Health Effects	16.61
Average Seasonal Adjustment	0.71
Average Seasonal Adjustment 1980 - 1990	1.01
Average Seasonal Adjustment 1990 - Present	0.96

Table 1: Descriptive statistics for United States beef consumption.

Lag	Mutual Information Function	Autocorrelation Function
1	2.7467933	0.93398562
2	2.3684692	0.88915847
3	2.1710312	0.89276679
4	2.1587009	0.89938676
5	2.2257604	0.84142258
6	1.9771303	0.7980301
7	1.7989259	0.80056196
8	1.7800623	0.79877481
9	1.7700694	0.73660526
10	1.5566994	0.68506057
11	1.3914136	0.68480521
12	1.3720171	0.67389864
13	1.3028482	0.61758272
14	1.1616996	0.57791756
15	1.0466497	0.59158978
16	1.016291	0.59141034
17	0.95244117	0.54004567
18	0.83317394	0.50942787
19	0.7706576	0.51685105
20	0.74752746	0.52364019
21	0.71221162	0.4736557
22	0.63290224	0.4465524
23	0.56110396	0.45503268
24	0.54848919	0.46283112
25	0.53868671	0.41268237
26	0.47839858	0.39274703
27	0.44433093	0.39660277
28	0.46024415	0.39379443
29	0.45976973	0.34909124
30	0.40708755	0.32961898

Table 2: The Mutual Information Function and Autocorrelation Function for the United States Beef time series.

Shapiro-Francia W' test for normal data				
Variable	**W'**	**V'**	**z**	**Prob>z**
ss x(t)[a]	0.89652	3.632	2.345	0.00951
ss x(t-4)	0.9006	3.489	2.274	0.0115
wf x(t)[b]	0.91934	2.831	1.899	0.02875
wf x(t-4)	0.89408	3.718	2.387	0.0085

[a]x(t) of the spring-summer cluster

[b]x(t) of the winter-fall cluster

Table 3: Shapiro-Francia test for normality.

Season	Year	X(t)	X(t-4)	EDM[a]	Z-Score	Chi-Square Prob.
	1991	15.945661	16.546138	0.421758	1.316159	0.482155
	1992	16.316898	15.945661	0.285082	0.590954	0.255823
	1993	15.776404	16.316898	0.400798	1.172680	0.443640
	1994	16.094410	15.776404	0.406421	1.422782	0.509039
	1995	16.141042	16.094410	0.085017	0.062166	0.030605
	1996	16.824933	16.141042	0.673383	3.522611	0.828180
Winter	**1997**	16.012675	16.824933	0.661268	3.464153	0.823083
	1998	16.362103	16.012675	0.267274	0.507738	0.224207
	1999	16.333423	16.362103	0.257597	0.603198	0.260365
	2000	16.696844	16.333423	0.565827	2.697202	0.740397
	2001	16.074835	16.696844	0.524009	2.210415	0.668858
	2002	16.196467	16.074835	0.112691	0.096207	0.046965
	2003	16.184524	16.196467	0.036565	0.011791	0.005878
	2004	15.962037	16.184524	0.190654	0.284106	0.132425
	2005	15.637469	15.962037	0.558816	2.699260	0.740664
	1991	16.142064	16.402338	0.223948	0.413962	0.186965
	1992	15.803769	16.142064	0.350743	0.994420	0.391775
	1993	15.807419	15.803769	0.509585	2.363850	0.693312
	1994	16.284576	15.807419	0.393981	1.191793	0.448932
	1995	16.119353	16.284576	0.111033	0.095508	0.046632
Fall	**1996**	16.006584	16.119353	0.157593	0.214148	0.101541
	1997	15.931499	16.006584	0.280158	0.706337	0.297541
	1998	16.325513	15.931499	0.301624	0.661895	0.281757
	1999	16.380071	16.325513	0.270768	0.652772	0.278473
	2000	16.415295	16.380071	0.331034	0.985429	0.389034
	2001	16.397065	16.415295	0.340249	1.051052	0.408756
	2002	16.633206	16.397065	0.527913	2.422256	0.702139
	2003	14.954773	16.633206	1.281180	11.962434	0.997474
	2004	16.287223	14.954773	1.231247	12.323405	0.997891
	2005	16.529866	16.287223	0.392581	1.299319	0.477776

[a]Euclidean distance from the cluster mean

Table 4: Chi-Square test based on the Confidence Ellipsoid for the winter/fall cluster.

The Chi-Square probability indicates which confidence ellipsoid each trajectory lies on.

Season	Year	X(t)	X(t-4)	EDM[a]	Z-Score	Chi-Square Prob.
Spring	**1991**	16.993437	17.365008	0.297169	0.985338	0.389007
	1992	16.862324	16.993437	0.197613	0.358899	0.164270
	1993	16.079414	16.862324	0.986747	9.758110	0.992396
	1994	16.715625	16.079414	1.045468	9.826020	0.992650
	1995	16.898459	16.715625	0.384896	1.300019	0.477959
	1996	17.404936	16.898459	0.400891	2.053637	0.641855
	1997	16.994627	17.404936	0.336416	1.250123	0.464771
	1998	16.856522	16.994627	0.202490	0.379470	0.172822
	1999	17.369811	16.856522	0.391008	1.991922	0.630632
	2000	17.100304	17.369811	0.303058	0.871618	0.353259
	2001	16.790013	17.100304	0.255154	0.752708	0.313641
	2002	17.509160	16.790013	0.544351	3.844278	0.853706
	2003	16.904258	17.509160	0.458731	2.481029	0.710765
	2004	16.886468	16.904258	0.229922	0.438186	0.196753
	2005	16.910961	16.886468	0.228155	0.433193	0.194745
Summer	**1991**	17.493748	17.401650	0.557880	2.653102	0.734609
	1992	17.120359	17.493748	0.428573	1.749381	0.583009
	1993	16.897662	17.120359	0.153713	0.292977	0.136264
	1994	17.189739	16.897662	0.227565	0.672364	0.285507
	1995	17.407351	17.189739	0.382295	1.400285	0.503486
	1996	16.887620	17.407351	0.369758	1.680941	0.568492
	1997	16.740252	16.887620	0.355045	1.101246	0.423409
	1998	17.132646	16.740252	0.343591	1.367740	0.495340
	1999	17.420007	17.132646	0.381241	1.494051	0.526226
	2000	17.546350	17.420007	0.611384	3.208538	0.798964
	2001	17.030634	17.546350	0.474422	2.357506	0.692338
	2002	17.364519	17.030634	0.323584	1.218907	0.456352
	2003	16.878534	17.364519	0.335790	1.415882	0.507342
	2004	16.921698	16.878534	0.228759	0.439024	0.197089
	2005	17.000673	16.921698	0.156413	0.223508	0.105736

[a]Euclidean distance from the cluster mean

Table 5: Chi-Square test based on the Confidence Ellipsoid for the spring/summer cluster. The Chi-Square probability indicates which confidence ellipsoid each trajectory lies on.

Figures

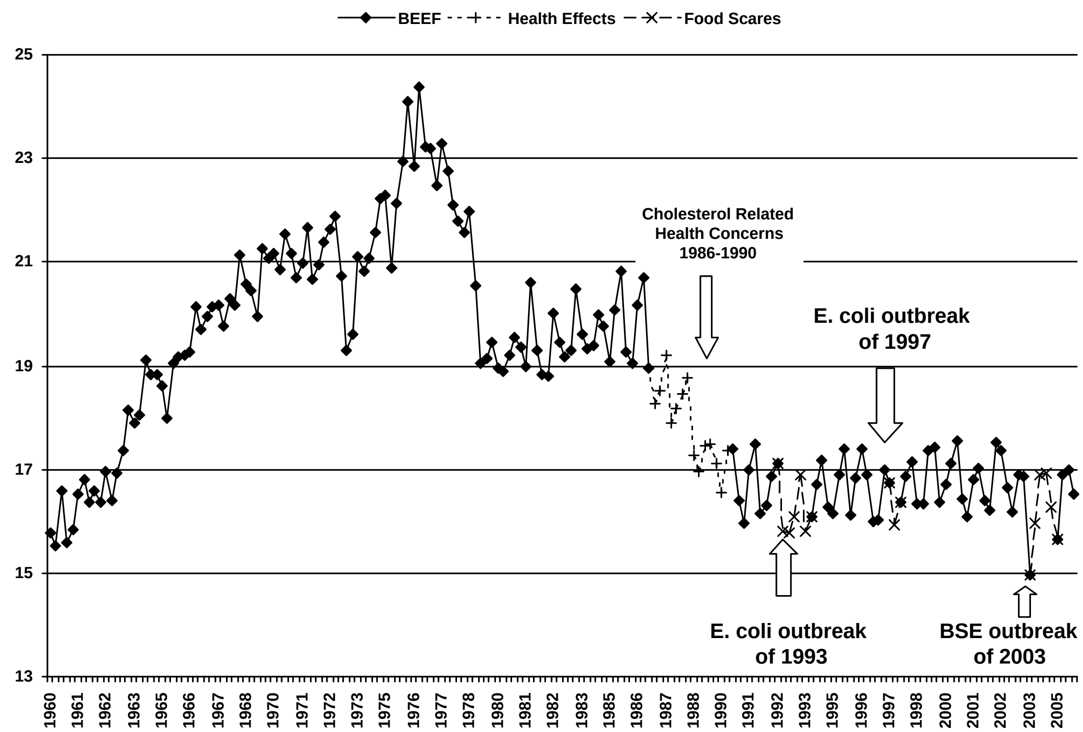

Figure 1: United States consumption of beef with indicated Health Effects and Food Scares; 1960-2005.

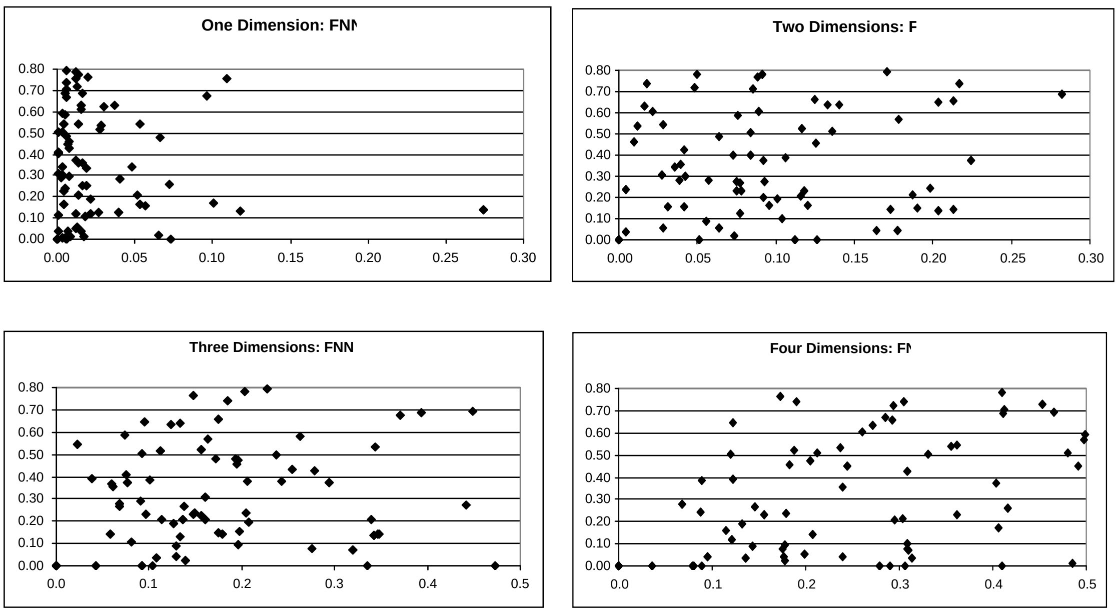

Figure 2: Graphical false nearest neighbors test for minimum embedding dimension indication a dimension of $\lambda = 2$.

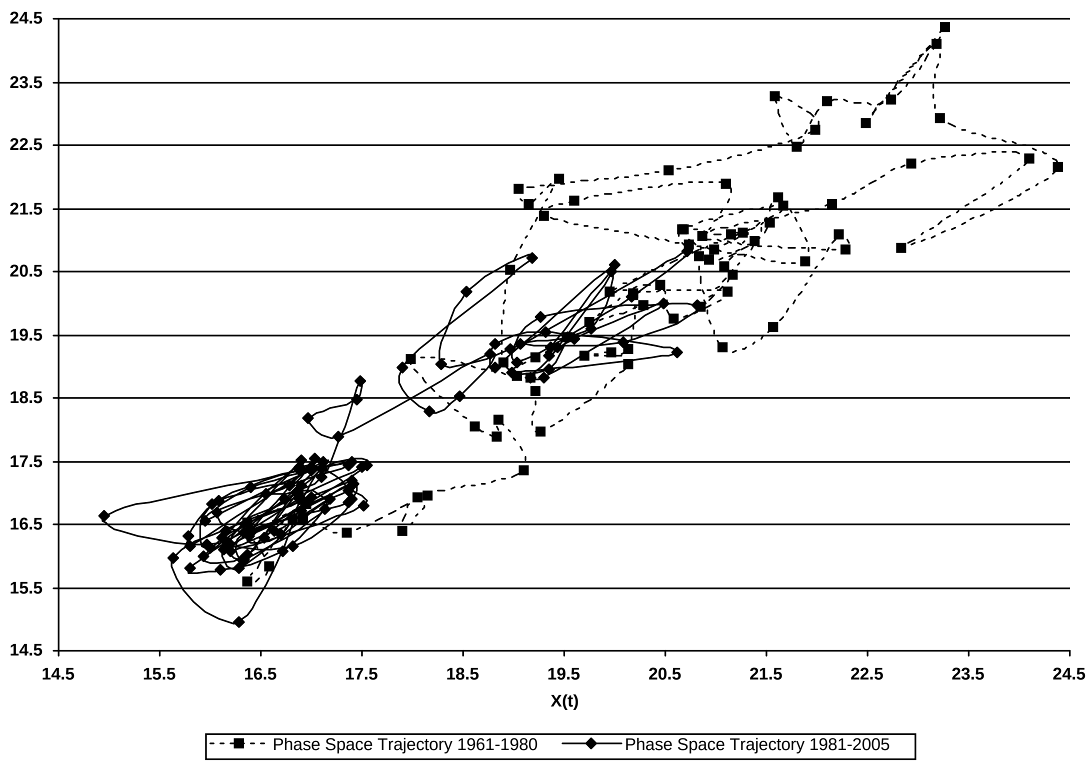

Figure 3: Reconstructed phase space for U.S. beef consumption; 1960-2005.

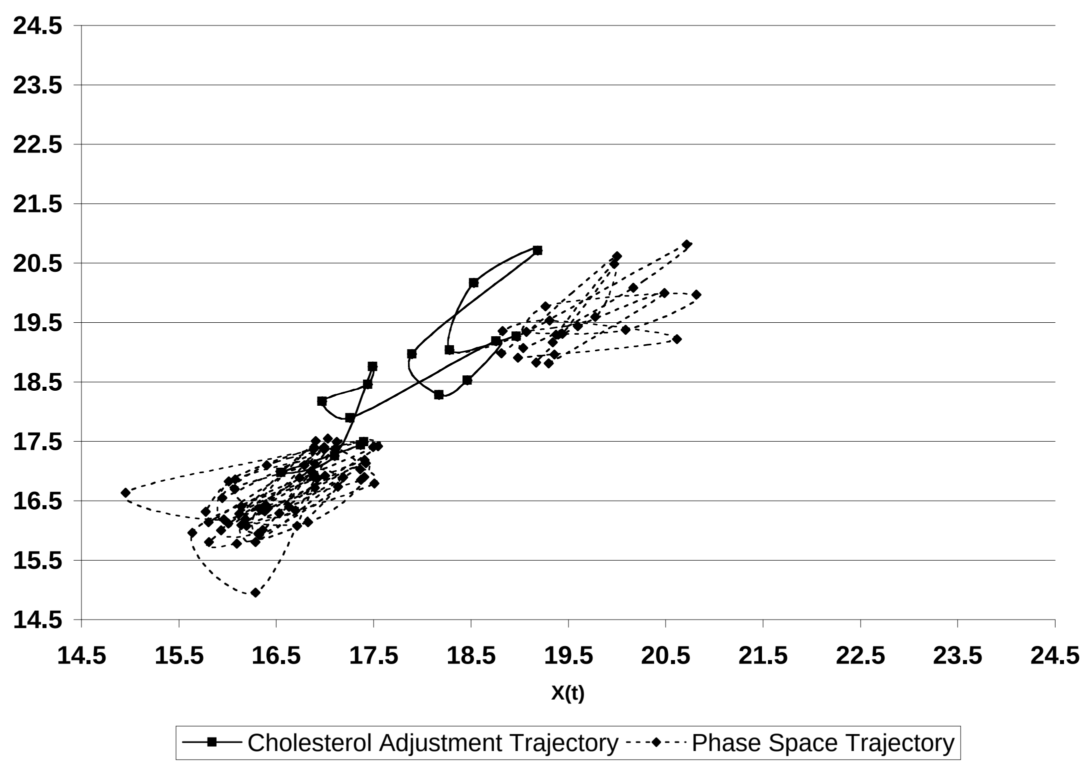

Figure 4: The effects of cholesterol on U.S. Beef consumption; 1980-2005.

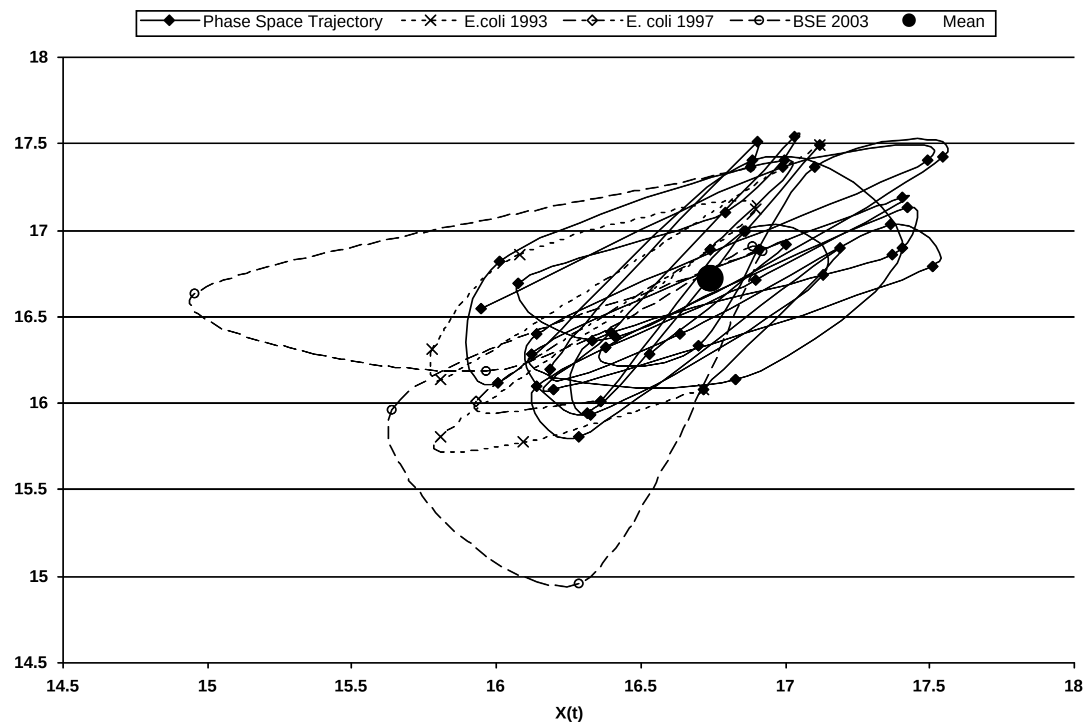

Figure 5: Reconstructed phase space with the effects of cholesterol controlled for; 1990-2005.

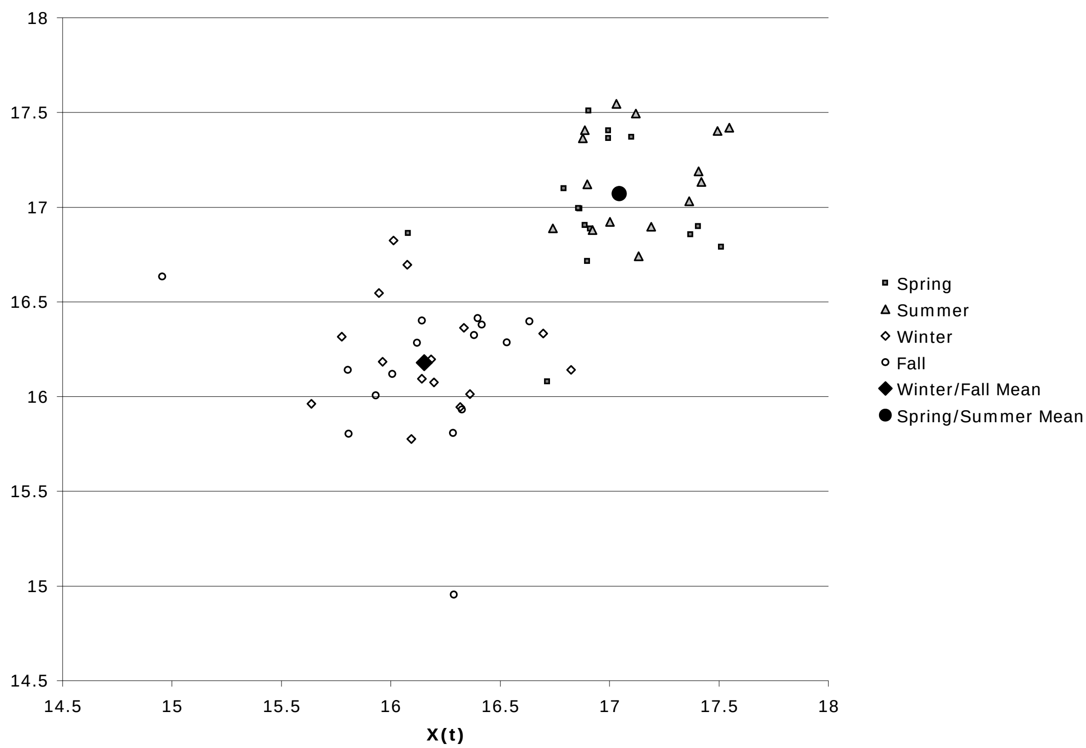

Figure 6: Reconstructed phase space indicating the seasonal clusters.

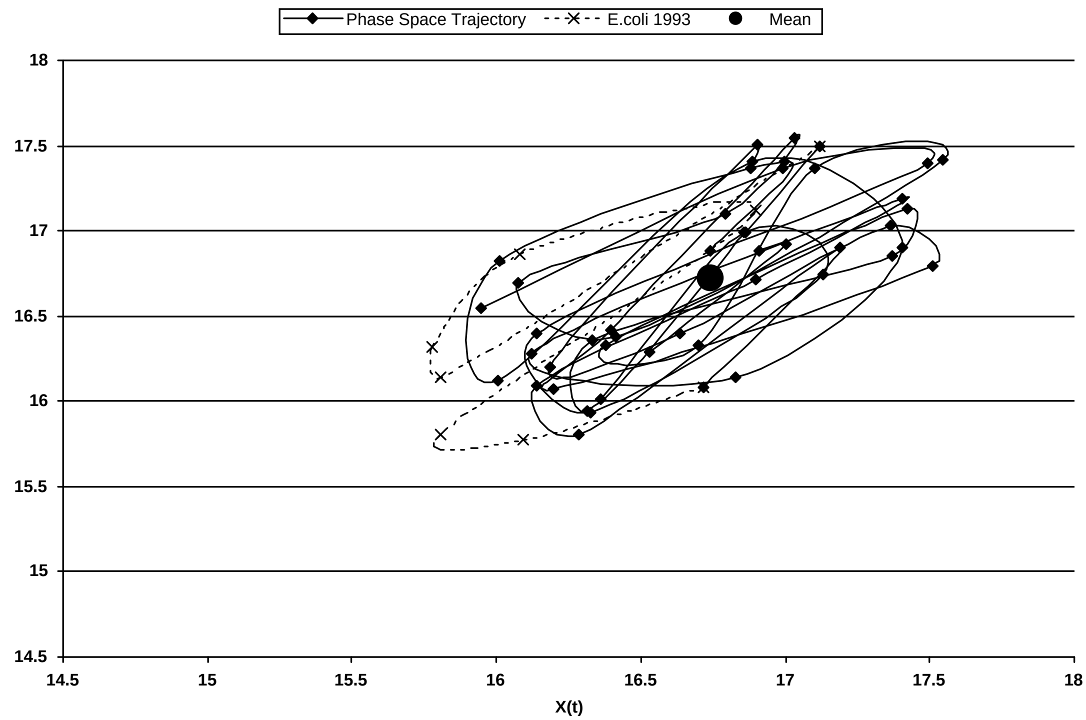

Figure 7: Reconstructed phase space isolating the effects of the Jack-in-the-Box E. coli outbreak of 1993; 1990-2005 (without other E. coli and BSE food scares).

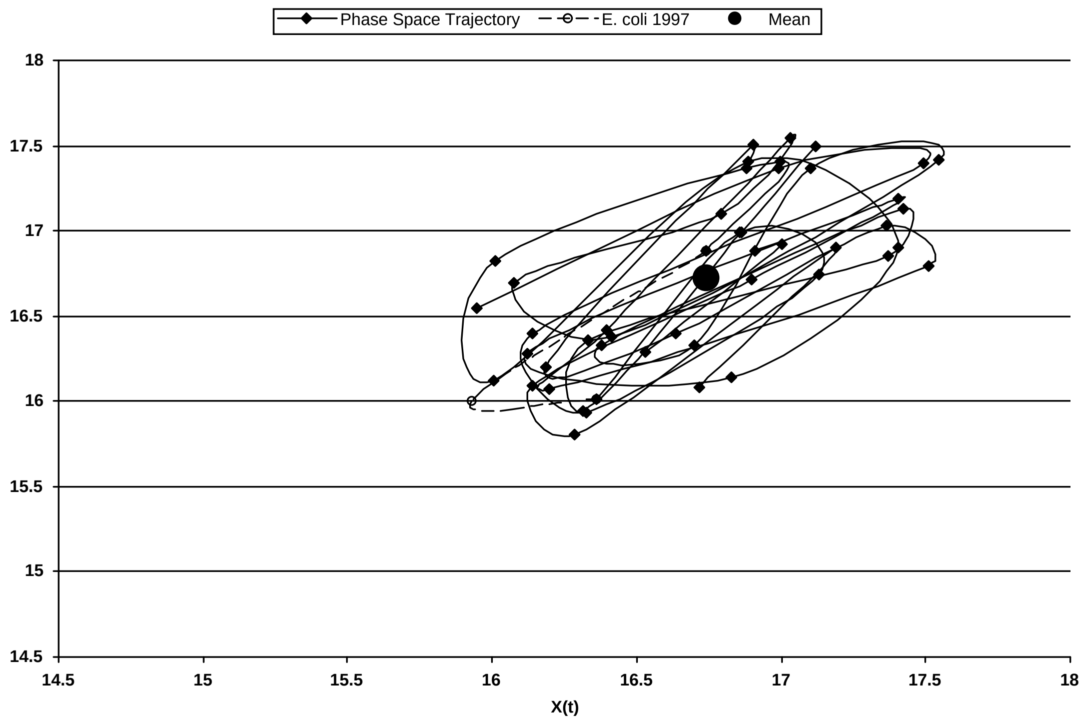

Figure 8: Reconstructed phase space isolating the effects of the Hudson Beef E. coli outbreak of 1997; 1990-2005 (without other E. coli and BSE food scares).

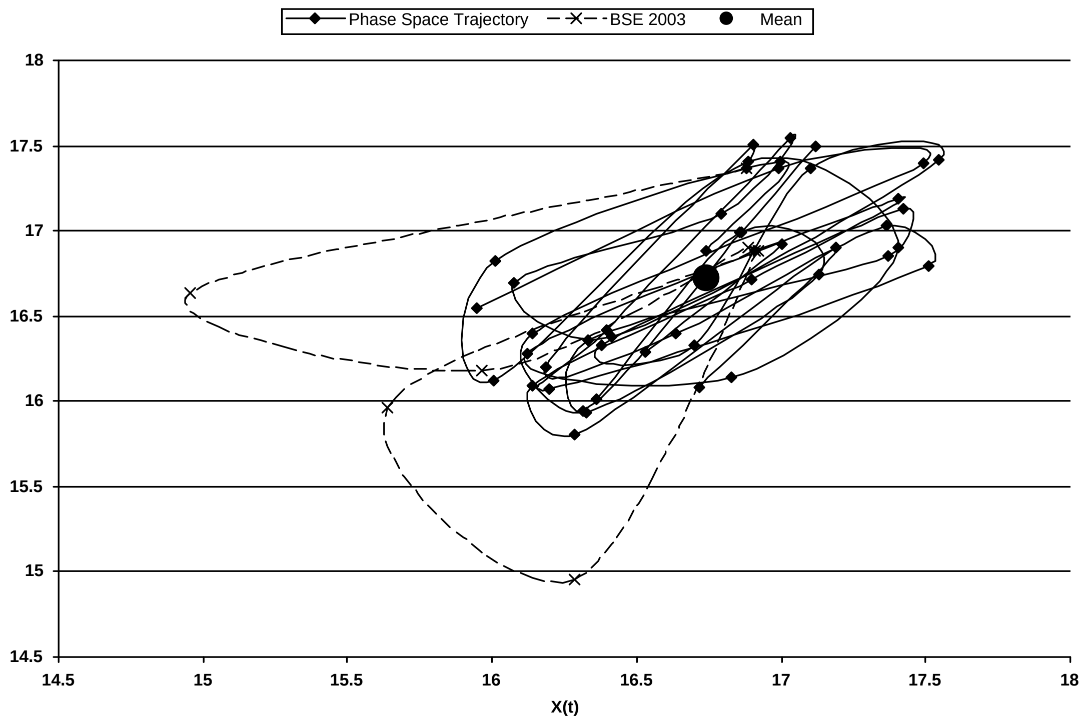

Figure 9: Reconstructed phase space isolating the effects of the BSE outbreak in the state of Washington in 2003; 1990-2005 (without E. coli food scares).

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